

# SOME STUDIES ON APPROXIMATE METHODS IN ACOUSTIC SCATTERING PROBLEMS

THESIS SUBMITTED FOR THE DEGREE OF  
DOCTOR OF PHILOSOPHY (SCIENCE)

OF WEST BENGAL UNIVERSITY OF TECHNOLOGY

RATAN KUMAR SAHA



MICROELECTRONICS DIVISION  
SAHA INSTITUTE OF NUCLEAR PHYSICS  
1/AF, BIDHANNAGAR, KOLKATA-700064.

November, 2006

# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Basics of acoustic wave and scattering</b>	<b>7</b>
2.1	Wave propagation in a homogeneous fluid medium . . . . .	7
2.2	The Wave equation . . . . .	8
2.2.1	Conservation of mass . . . . .	8
2.2.2	Conservation of momentum . . . . .	10
2.2.3	Equation of state . . . . .	11
2.2.4	Derivation of the wave equation . . . . .	12
2.2.5	Solutions of the wave equation . . . . .	14
2.2.6	Reflection and Transmission of plane wave . . . . .	16
2.3	Scattering of plane wave . . . . .	18
2.3.1	Scattering amplitude in the exact method . . . . .	19
2.4	Radiation from a circular transducer . . . . .	24
<b>3</b>	<b>Scattering solutions by approximate methods</b>	<b>31</b>
3.1	Wave equation in an inhomogeneous medium . . . . .	34
3.2	Scattering solution by approximate methods . . . . .	36
3.2.1	Scattering amplitude: Integral equation approach . . . . .	36
3.2.2	Born approximation . . . . .	39

---

3.2.3	Modified Born approximation . . . . .	41
3.2.4	Alternative derivation of Modified Born approximation . .	42
3.2.5	Distorted wave Born approximation . . . . .	43
3.2.6	Eikonal approximation . . . . .	44
3.3	Evaluation of validity domains of approximations . . . . .	48
3.4	Conclusions . . . . .	57
<b>4</b>	<b>Scattering of pulsed plane wave</b>	<b>59</b>
4.1	Formulas for scattering of a pulsed plane wave . . . . .	59
4.2	Numerical comparison . . . . .	63
4.3	Conclusions . . . . .	68
<b>5</b>	<b>Particle sizing</b>	<b>70</b>
5.1	Scattering by a sphere . . . . .	72
5.1.1	Plane wave scattering . . . . .	72
5.1.2	Pulsed plane wave scattering . . . . .	74
5.2	Scattering by a cylinder . . . . .	75
5.2.1	Plane wave scattering . . . . .	75
5.2.2	Pulsed plane wave scattering . . . . .	77
5.3	Numerical comparison . . . . .	78
5.4	Conclusions . . . . .	84
<b>6</b>	<b>Development of an experimental facility</b>	<b>87</b>
6.1	Scattering by a long rigid cylinder . . . . .	87
6.2	Development of the system . . . . .	90
6.2.1	Description of the container . . . . .	90
6.2.2	Description of the angular positioning system . . . . .	91
6.2.3	Description of ambient medium and various gadgets . . . .	93
6.3	Measurements and experimental results . . . . .	95

---

6.3.1	Reliability assessment of the instrument . . . . .	95
6.3.2	Line scan results . . . . .	98
6.3.3	Angular scan results . . . . .	99
6.4	Conclusions . . . . .	101
<b>7</b>	<b>Conclusion</b>	<b>104</b>
	<b>Appendices</b>	<b>109</b>
<b>A</b>	<b>Scattering by a sphere</b>	<b>110</b>
A.1	Long wavelength approximation . . . . .	110
A.2	Scattered wave in approximate methods . . . . .	111
A.3	Modified Born approximation . . . . .	112
A.4	Eikonal approximation . . . . .	114
A.5	Derivation of the Born approximation result from the Eikonal approximation . . . . .	116
<b>B</b>	<b>Scattering by a long cylinder</b>	<b>118</b>
B.1	Scattering amplitude in exact method . . . . .	118
B.1.1	Long wavelength approximation . . . . .	121
B.2	Scattering amplitude in approximate methods . . . . .	122
B.2.1	Born approximation . . . . .	122
B.2.2	Modified Born approximation . . . . .	123