

Some Applications of Hard Thermal Loop Perturbation Theory in Quark Gluon Plasma

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DECLARATION

I, hereby declare that the investigation presented in the thesis has been carried out by me. The work is original and has not been submitted earlier as a whole or in part for a degree / diploma at this or any other Institution / University.



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List of Publications arising from the thesis

Journal

1. Three-loop HTLpt thermodynamics at finite temperature and chemical potential, *Najmul Haque, Aritra Bandyopadhyay, Jens O. Andersen, Munshi G. Mustafa, Michael Strickland and Nan Su*, **JHEP** **1405**, **027** (2014).
2. Three-loop HTLpt Pressure and Susceptibilities at Finite Temperature and Density, *Najmul Haque, Jens O. Andersen, Munshi G. Mustafa, Michael Strickland, Nan Su*, **Phys. Rev. D** **89**, **061701(R)** (2014) .
3. Quark Number Susceptibilities from Two-Loop Hard Thermal Loop Perturbation Theory, *Najmul Haque, Munshi G. Mustafa, Michael Strickland*, **JHEP** **1307**, **184** (2013).
4. Two-loop HTL pressure at finite temperature and chemical potential, *Najmul Haque, Munshi G. Mustafa, Michael Strickland*, **Phys. Rev. D** **87**, **105007** (2013).
5. Conserved Density Fluctuation and Temporal Correlation Function in HTL Perturbation Theory, *Najmul Haque, Munshi G. Mustafa, Markus H. Thoma*, **Phys. Rev. D** **84**, **054009** (2011).
6. Low Mass Dilepton Rate from the Deconfined Phase, *Carsten Greiner, Najmul Haque, Munshi G. Mustafa, Markus H. Thoma*, **Phys. Rev. C** **83b**, **014908** (2011).

Chapters in books and lectures notes

N. A.

Conferences

- Quark Number Susceptibility and Thermodynamics in HTL approximation
Najmul Haque, Munshi G. Mustafa,
Nucl. Phys. A 862-863, 271 (2011).

Others

- A Modified Hard Thermal Loop Perturbation Theory
Najmul Haque, Munshi G. Mustafa
arXiv: 1007.2076 [hep-ph].


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DEDICATIONS

Dedicated to my daughter.

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