

Lagrangian Fluid Technique to Study Nonlinear Plasma Dynamics

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by

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ABSTRACT

A theoretical study to explore the dynamics of large-amplitude normal modes of a cold magnetized electron-ion plasma is presented in this thesis by using a Lagrangian fluid technique. A fruitful application of such a technique is shown to be instrumental in obtaining (exact) explicit space-time dependent solutions, especially in double species wave dynamics problems. The obtained large-amplitude solutions are shown to be capable of characterizing two interesting singular wave phenomena in plasmas, viz., wave-breaking and wave-collapse. These phenomena are singular in a sense that, at the wave-breaking or wave-collapse event, field variables lose their initial smoothness leading to the formation of singularities at a finite time. ‘Phase-mixing’ phenomenon is seen as an underlying physical phenomenon which causes wave-breaking at arbitrary amplitudes. A number of physical processes which may induce mixing of phases of different parts of a wave have been discussed in detail. An inhomogeneity in the equilibrium magnetic field is identified as a novel source of phase-mixing in the excited ‘electrostatic’ normal modes of a magnetized plasma. The results of our investigations on wave-breaking in magnetized plasmas will be of relevance to particle energization and plasma heating in laboratory experiments and space plasma situations. On the other hand, based on exact nonstationary solutions, the collapse behavior of magnetosonic waves in a low-beta collisional magnetoplasma has been predicted. The relevance of our investigation of the magnetosonic wave-collapse process has been highlighted in the early stage of magnetic star formation.

LIST OF PUBLICATIONS

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