

**SOME ASPECTS OF QUANTUM MECHANICS
AND FIELD THEORY ON
NONCOMMUTATIVE SPACES**

**THESIS SUBMITTED TO
JADAVPUR UNIVERSITY
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY (SCIENCE)**

By

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2008

TABLE OF CONTENTS

	Page
Acknowledgements	vi
Abstract	vii
List of Figures	viii
Chapters:	
1. Introduction	1
1.1 Motivation and Overview of the Thesis	1
1.2 Organization of the Thesis	8
Bibliography	10
2. Brief Introduction to Noncommutative Theories	13
2.1 Landscape of Theories on Noncommutative Spaces	13
2.2 Weyl-Wigner Correspondence	14
2.3 Groenewold-Moyal Product	17
2.4 Deformation of Classical and Quantum Theories	20
2.4.1 Classical and Quantum Mechanics	20
2.4.2 Field Theories	22
2.4.3 Bounds on the Noncommutative Parameter	27
Bibliography	28
3. Noncommutative Quantum Mechanics in Phase Space	32
3.1 Introduction	32
3.2 Classical Domain	34
3.3 Quantum Domain	35

3.3.1	Deformed Products	36
3.3.2	Quantum Condition	37
3.3.3	Baker's Construction of Integral Equation	38
3.3.4	Iterated Quantum Conditions and Integral Equations	40
3.3.5	Generic Form of Density Distribution	42
3.4	Some Consequences of the Generic Form of Density Distribution	46
3.4.1	Weyl-Correspondence	46
3.4.2	Quantum and Normalization Conditions Preserving Variations	50
3.5	Dynamical Equations	51
3.5.1	Generalized Moyal Equation of Motion	51
3.5.2	Generalized Heisenberg Equation in Phase Space	52
3.5.3	Time dependent Schrödinger Equation	52
3.5.4	Heisenberg Equation	53
3.6	Eigenvalue Equations	53
3.6.1	*-genvalue Equation in Phase Space	54
3.6.2	The Case in Hilbert Space	59
3.7	Twisted Galilean Symmetry	60
3.8	Summary	61
	Bibliography	63
4.	Noncommutative Oscillators and the Commutative Limit	65
4.1	Introduction	65
4.2	Harmonic Oscillator	65
4.3	Anharmonic Oscillator	68
4.4	Summary	70
	Bibliography	71
5.	Noncommutative Schrödinger Field Theory	72
5.1	Introduction	72
5.2	Noncommutative Klein-Gordon Theory	74
5.3	The Newton-Cartan Spacetime Structures	78
5.4	The Magnetic Limit	80
5.5	Summary	86
	Bibliography	88
6.	Summary, Conclusions and Future Outlook	90
	Bibliography	94

Appendix:

A.	Twisted Space-time Symmetries	95
A.1	Introduction	95
A.2	Twisted Poincaré Transformation	95
A.3	Twisted Galilean Transformation	99
	Bibliography	100
	List of Publications	101