was found to be always in the right direction. When the filament lamp was used as the source of light, all irregularities due to the variation of the source of light vanished. As soon as light is struck, the spot of light slowly creeps up towards the new position of equilibrium about which it oscillates in accordance with the equation (i).

Ultimately the oscillation dies away and the spot becomes quite steady, which could be maintained for 15 minutes (we did not try to keep the spot steady for a greater length of time because the tungsten filaments, being kept in a horizontal position, are gradually deformed on account of their plasticity at the high temperature within the lamp).

In one set of experiments one of the vanes was silvered while the other consisted of two clear pieces of microscopic cover glass. We found that when light was allowed to fall on the clear glass surface there was practically no deflection. In another set of experiments one of the vanes was silvered and the other was lampblacked. It was found that generally if the source of light was not too intense, the deflection of the black surface was approximately one half of that of the silvered one. If the source of light was very intense so much heat was absorbed that the junctions (which were all of shellac) melted off. Quantitative experiments were therefore impossible with that surface.

One of the results of our quantitative experiments is given below:—

Mean deflection (mean of several experiments)	=28·5 Divns.
Distance of the scale from the mirror	=100 cm
Distance "d" of the plane of the filament from the diaphragm	=73 cm

Therefore the upper limit of the total theoretical pressure (without allowing for absorption or reflexion) is equal to

$$2\times\frac{6\cdot6\times220\times10^{7}\times(3\cdot25)^{2}}{4\times73^{2}\times3\times10^{10}}=4\cdot8\times10^{-4}\text{ dynes} \qquad \text{(A)}.$$

The pressure calculated from deflections is equal to

$$2\cdot3\times10^{-5}\times14\cdot25=3\cdot33\times10^{-4}\text{ dynes} \qquad \text{(B)}.$$

The observed pressure is about 70 per cent of the expression (A), which is the pressure calculated on the supposition that the whole amount of energy given out by the filament is freely transmitted by the various glass media, and is totally reflected by the silvered surface. As a matter of fact, none of these assumptions is correct. If T is the fraction of total energy transmitted by thick glass, and ρ be the reflecting power of a silver glass-surface the actual pressure should be

$$P_0\frac{T}{2}(1+\rho)\ (1-\epsilon)$$

where P_0 is the quantity (A).

According to the experiments of Rubens and Hagen[7] $\rho=90\cdot5\%$ unfortunately no data is available for the transmission coefficient, but on account of the preponderance of rays of short wave length in the spectrum of the light from a tungsten filament, it cannot be less than 80%.

Considering these facts, we are probably justified in asserting that the agreement between observed and theoretical values is at least qualitatively quite good. On a future occasion we hope to return to the problem of a rigorous quantitative determination of total incident energy.

In conclusion, we beg to record our best thanks to Prof. C. V. Raman, and the teaching staff of the University College of Science, for the interest they have taken in the work; and to Mr. N. Basu, B.Sc., for much useful help.

[7]Obtained by extrapolation from the data of Rubens and Hagen on the supposition that the maximum emission of energy from a tungsten filament is at 1 μ [Kohlrausch, Praktische Physik, Tabellen].

5. ON THE DYNAMICS OF THE ELECTRON*

(*Phil. Mag., Sr. VI*, **36**, 76, 1918)

Mass as a fundamental physical concept has been introduced into Physics by Newton's Second Law of Motion, which may be said to form the corner-stone of classical Mechanics. But in spite of its splendid success, physicists have always encountered some difficulty in realising mass as a fundamental physical concept in the same sense as the concepts of time and space. The fundamental object of mechanics is to provide a scaffolding by means of which the motion of material bodies can be surveyed and followed, when these are subjected to various disturbing influences. Some hypothesis must be introduced for taking into account the influences of these disturbing agencies. The question is: "Are Newton's Second Law of Motion and the ideas underlying it quite sufficient for all possible cases of motion, or are we to search for some more general principle ?" Some physicists are in fact in favour of introducing Energy as a more fundamental physical concept than Mass, thereby basing the Science of Mechanics on various Energy-theorems.

So long as we hold to the principle of invariability of

*Communicated by Prof. A. W. Porter, F.R.S.

mass, there can of course be no question about the utility of the second law. But in the electron we have a physical entity which defies this limitation. If we want to survey its motion, and have no other means of doing so than classical mechanics, we must ascribe to it a certain mass, but for aught we know this mass is neither definite nor invariant during motion. Consequently the scaffolding which enables us to study and survey the motion of material (i.e. non-electrical) particles breaks down in this case. Some other system of Mechanics other than Newtonian must be formulated. In this attempt, we must remember that the electric charge is the only invariant physical quantity, consequently in place of mass, this quantity ought to appear in the equation of motion. We must also take cognisance of the newly discovered relations between time and space which are embodied in the Principle of Relativity.

I may be allowed to remark at this place that though the inadequacy of classical mechanics for studying the motion of electrons is now admitted on all hands, and many attempts are being made for formulating the exact dynamics of the electron,—the authors of many of these theories have not been able to rid themselves of the preconceived ideas of classical mechanics. I shall, in the first place, explain my own method, point out the characteristic features of my theory, and then compare it with other theories.

1.

The equations of motion of a material particle are derived from Newton's Second Law of Motion—rate of change of momentum is proportional to the force applied. Combining this principle with the principle of constancy of mass during motion, we obtain

$$m\frac{d^2x}{dt^2} = X,\ m\frac{d^2y}{dt^2} = Y,\ m\frac{d^2z}{dt^2} = Z.$$

The terms $m\frac{d^2x}{dt^2}, m\frac{d^2y}{dt^2}, m\frac{d^2z}{dt^2}$ are known as the components of the "Effective Force", and the law may be expressed by saying that the "Effective Force" is equivalent to the "Impressed Force".

In the case of the Electron, we hold to the axiom that the "Effective Force is equivalent to the Impressed Force". No *prima facie* reason can be given for the introduction of this hypothesis, just as in the case of the motion of material bodies. It is to be justified by its success in dealing with the problem at hand.

2.

The Impressed Force on the electron can be easily calculated with the aid of Lorentz's Theorem of Ponderomotive Force. If (X, Y, Z) be the components of the electric field, (L, M, N) be the components of the magnetic field at any point, and ρ be the density of electricity, the components of the force per unit volume at the point are

$$\left.\begin{aligned} f_x &= \rho\,[X+\frac{1}{c}(v_2 N - v_3 M)] \\ f_y &= \rho\,[Y+\frac{1}{c}(v_3 L - v_1 N)] \\ f_z &= \rho\,[Z+\frac{1}{c}(v_1 M - v_2 L)] \end{aligned}\right\},$$

(v_1, v_2, v_3) being the components of the velocity with which the charge moves.

The rate at which work is done is given by the equation

$$\begin{aligned} f_t &= f_x v_1 + f_y v_2 + f_z v_3 \\ &= \rho\,[X v_1 + Y v_2 + Z v_3]. \end{aligned}$$

In accordance with the ideas of the Principle of Relativity we can write the components of the force-four-vector in the form

$$\left.\begin{aligned} f_x &= \rho_0\,[\qquad\quad + f_{12} w_2 + w_3 f_{13} + w_4 f_{14}] \\ f_y &= \rho_0\,[f_{21} w_1 \qquad\quad + w_3 f_{23} + w_4 f_{24}] \\ f_z &= \rho_0\,[w_1 f_{31} + w_2 f_{32} \qquad\quad + w_4 f_{34}] \\ f_l &= \rho_0\,[w_1 f_{41} + w_2 f_{42} + w_3 f_{43} \qquad\quad] \end{aligned}\right\} \quad \ldots.(1)$$

these equations are obtained by writing[1]

f_{23}, f_{31}, f_{12} for L, M, N,

f_{14}, f_{24}, f_{34} for $-i\,(X, Y, Z)$,

w_1, w_2, w_3, w_4 for $\frac{1}{\sqrt{1-v^2/c^2}}\,[v_1/c, v_2/c, v_3/c, i]$,

ρ_0 for $\rho\sqrt{1-v^2/c^2}$.

For finding out the total force on the electron, we have to integrate the above expressions for the force-four-vector over the whole volume of the electron. Supposing that the components of the electric and magnetic force do not vary throughout the volume of the electron, the force-components are obtained by writing simply (e) the invariant charge instead of (ρ_0) in equations (1).

3.

The calculation of the Effective force is a matter of some difficulty. The question is: "If an electron moves with a variable velocity, what are the terms corresponding to the quantities $\left(m\frac{d^2x}{dt^2}, m\frac{d^2y}{dt^2}, m\frac{d^2z}{dt^2}\right)$ in particle dynamics?" Einstein solves the difficulty by saying that instead of the observer's time dt we have to introduce here the proper time (Eigenzeit) of motion of the electron. This conclusion[2] is reached in a general way from his theory of the equivalence of the forms for equation of motion of material particles when referred to systems moving with uniform

[1] The notation used throughout the paper is that of Minkowski, vide *Math. Ann.* Vol. lxviii, p. 472 *et seq.* § 12, where this particular theorem occurs in an abbreviated form.

[2] A. Einstein, *Jahrbuch der Radioaktivitat*, Vol. iv. 1907

velocity past each other. Minkowski[3] practically uses the same hypothesis as I have done (Effective force is equivalent to the Impressed force), but in case of the electron he begins by implicitly ascribing a rest-mass to the electron. But the method adopted by me is fundamentally different, as will appear in due course. Elsewhere, Minkowski[4] deduces it from the Principle of Least Action, combined with the principle of conservation of mass in a space perpendicular to the axis of motion. Besides, the investigation has a direct bearing on the theory of Electromagnetic momentum as developed by Lorentz and Abraham.

4.

Let us now concentrate our attention on a single electron moving with a velocity v. The force components at an external point due to the motion of the electron are given by the equations (1). Generalising or rather recasting Maxwell's theorem of stresses into new forms, Minkowski has shown that the force components (f_x, f_y, f_z, f_l) can be put into the forms

$$\left.\begin{aligned} f_x&=\frac{\partial X_x}{\partial x}+\frac{\partial X_y}{\partial y}+\frac{\partial X_z}{\partial z}+\frac{\partial X_l}{\partial l}\\ f_y&=\frac{\partial Y_x}{\partial x}+\frac{\partial Y_y}{\partial y}+\frac{\partial Y_z}{\partial z}+\frac{\partial Y_l}{\partial l}\\ f_z&=\frac{\partial Z_x}{\partial x}+\frac{\partial Z_y}{\partial y}+\frac{\partial Z_z}{\partial z}+\frac{\partial Z_l}{\partial l}\\ f_l&=\frac{\partial L_x}{\partial x}+\frac{\partial L_y}{\partial y}+\frac{\partial L_z}{\partial z}+\frac{\partial L_l}{\partial l}\end{aligned}\right\} \quad ..(2)$$

where
$$\left.\begin{aligned} X_x&=\frac{1}{8\pi}[f_{23}^2+f_{34}^2+f_{42}^2-f_{12}^2-f_{13}^2-f_{14}^2]\\ X_y&=\frac{1}{4\pi}[f_{13}f_{32}+f_{14}f_{42}].\end{aligned}\right\} .(3)$$

The theorem is proved by substituting, in equations (1), the values of $\rho_o w_1, \rho_o w_2, \rho_o w_3, \rho_o w_4$ obtained from the fundamental equation

$$\text{lor} f=4\pi\rho_o(w_1, w_2, w_3, w_4)$$

and effecting the necessary transformations with the aid of the second fundamental equation

$$\text{lor} f=O.^5$$

In the present case, the field is due to a single moving charge. The quantities $[X_x, X_y, \ldots]$ can be easily calculated from the Potential four-vector $\mathbf{a}$, for the six-vector f is equivalent to curl $\mathbf{a}$.

In a paper[5] communicated sometime ago to the *Philosophical Magazine*, I have shown that the Potential four-vector $\mathbf{a}$ at an external space-time point (x', y', z', l') due to the motion of a charge e occupying the point (x, y, z, l) is equivalent to $\frac{ew}{R}$, where

$$w \text{ is velocity four-vector } = \left(\frac{dx}{ds}, \frac{dy}{ds}, \frac{dz}{ds}, \frac{dl}{ds}\right),$$

and R is the perpendicular distance from the external point on the line of motion of the electron. We have

$$\begin{aligned} R^2=&(x-x')^2+(y-y')^2+(z-z')^2+(l-l')^2\\ &+[(x-x')\,w_1+(y-y')\,w_2+(z-z')\,w_3+(l-l')\,w_4]^2.\end{aligned}$$

We have now

$$f_{hk}=\frac{\partial \mathbf{a}_k}{\partial x_h}-\frac{\partial \mathbf{a}_h}{\partial x_k} \qquad (h, k=1, 2, 3, 4),$$

$$\begin{aligned}\therefore f_{12}=\frac{\partial \mathbf{a}_2}{\partial x'}-\frac{\partial \mathbf{a}_1}{\partial x'}&=\frac{\partial}{\partial x'}\left(\frac{ew_2}{R}\right)-\frac{\partial}{\partial y'}\left(\frac{ew_1}{R}\right)\\ &=e\,(\alpha_1 w_2-\alpha_2 w_1);\end{aligned}$$

where

$$\alpha_1=\frac{\partial}{\partial x'}\left(\frac{1}{R}\right), \ \alpha_2=\frac{\partial}{\partial y'}\left(\frac{1}{R}\right), \ \alpha_3=\frac{\partial}{\partial z'}\left(\frac{1}{R}\right),$$
$$\alpha_4=\frac{\partial}{\partial l'}\left(\frac{1}{R}\right).$$

Therefore we have

$$\begin{aligned} X_x=\frac{e^2}{8\pi}[&(\alpha_2\omega_3-\alpha_3\omega_2)^2+(\alpha_3\omega_4-\alpha_4\omega_3)^2+(\alpha_4\omega_2-\alpha_2\omega_4)^2\\ &-(\alpha_1\omega_2-\alpha_2\omega_1)^2-(\alpha_1\omega_3-\alpha_3\omega_1)^2-(\alpha_1\omega_4-\alpha_4\omega_1)^2].\end{aligned}$$

Now putting $\quad \alpha^2=\alpha_1^2+\alpha_2^2+\alpha_3^2+\alpha_4^2$

and using the identity

$$\alpha_1\omega_1+\alpha_2\omega_2+\alpha_3\omega_3+\alpha_4\omega_4=0,$$

we easily prove that

$$X_x=\frac{e^2}{8\pi}\left[-\alpha^2\,(1+2\omega_1^2)+2\alpha_1^2\right].$$

Similarly

$$Y_y=\frac{e^2}{8\pi}\left[-\alpha^2\,(1+2\omega_2^2)+2\alpha_2^2\right],$$

$$X_y=\frac{e^2}{4\pi}\left[-\omega_1\omega_2\,\alpha^2+\alpha_1\alpha_2\right], \ \&c.$$

We shall now calculate the total force on the space exterior to the electron. According to the Principle of Relativity, this space must be uniquely defined. In our case, this space is perpendicular to the axis of motion of the electron, and is bounded on the inside by the surface of the electron. The external boundary is at an infinite distance. Let $d\Omega$ denote an element of volume of this space. Then the total force is given by

$$F_x=\int f_x\,d\Omega=\int\left[\frac{\partial X_x}{\partial x'}+\frac{\partial X_y}{\partial y'}+\frac{\partial X_z}{\partial z'}+\frac{\partial X_l}{\partial l'}\right]d\Omega \qquad (4)$$

Now since a, and consequently $f_{12}, f_{23} \ldots f_4 \ldots$, are functions of the relative distance $[(x-x'), (y-y'), (z-z'), (l-l')]$,

we have

$$\frac{\partial X_x}{\partial x'}=-\frac{\partial X_x}{\partial x},$$

[3]H. Minkowski, "Raum und Zeit," § iv. *Phys. Zeit.*, 1911.
[4]H. Minkowski, *Math. Ann.*, vol. lxviii. Appendix.
[5]It seems to have escaped the notice of investigators on this particular subject that the Potential four-vector in the form given by me is implicitly contained in a statement of Minkowski's ("Raum und Zeit," §5). The passage came to my notice only recently when I was making a critical study of Minkowski's works.

Therefore

$$F_x = -\left[\frac{\partial}{\partial x}\int Xx\,d\Omega + \frac{\partial}{\partial y}\int Xy\,d\Omega + \frac{\partial}{\partial z}\int Xz\,d\Omega + \frac{\partial}{\partial l}\int X_l\,d\Omega\right].$$

Now

$$\int X_x\,d\Omega = \frac{e^2}{8\pi}\int\left[-\alpha^2(1+2\,\omega_1^2)+2\,\alpha_1^2\right]d\Omega,$$

$$\int X_y\,d\Omega = \frac{e^2}{4\pi}\int\left[-\omega_1\omega_2\,\alpha^2+\alpha_1\alpha_2\right]d\Omega, \text{ \&c.}$$

We have now to calculate the value of the integrals $\int \alpha_1^2\,d\Omega$, $\int \alpha_2^2\,d\Omega$, $\int \alpha_1\alpha_2\,d\Omega$, &c.

We have

$$\alpha_1 = \frac{\partial}{\partial x'}\left(\frac{1}{R}\right) = \frac{1}{R^3}\left[(x-x')+\omega_1\left\{(x-x')\omega_1+(y-y')\omega_2+(z-z')\omega_3+(l-l')\omega_4\right\}\right].$$

Now let us introduce a Lorentz-transformation by means of which the axis of motion becomes the new time-axis. Let (ξ, η, ζ, ν) denote the new coordinates. We have then

$$\begin{bmatrix}(\xi-\xi')\\(\eta-\eta')\\(\zeta-\zeta')\\(\nu-\nu')\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12} & A_{13} & A_{14}\\ A_{21} & A_{22} & A_{23} & A_{24}\\ A_{31} & A_{32} & A_{33} & A_{34}\\ A_{41} & A_{42} & A_{43} & A_{44}\end{bmatrix}\begin{bmatrix}x-x'\\y-y'\\z-z'\\l-l'\end{bmatrix},$$

$$\left.\begin{aligned}&\text{where } A_{1k}^2+A_{2k}^2+A_{3k}^2+A_{4k}^2=1\\ &\text{and } A_{1h}A_{1k}+A_{2h}A_{2k}+A_{3h}A_{3k}+A_{4h}A_{4k}=0\end{aligned}\right\}.$$

Since the line of motion is the new time-axis, we have

$$(\nu-\nu') = i[\omega_1(x-x')+\omega_2(y-y')+\omega_3(z-z')+\omega_4(l-l')];$$

we have therefore

$$A_{41}=i\omega_1,\quad A_{42}=i\omega_2,\quad A_{43}=i\omega_3,\quad A_{44}=i\omega_4.$$

Now using the above transformation, we have

$$R^2=(\xi-\xi')^2+(\eta-\eta')^2+(\zeta-\zeta')^2$$

and

$$\begin{aligned}&(x-x')+\omega_1[(x-x')\omega_1+(y-y')\omega_2+(z-z')\omega_3+(l-l')\omega_4]\\ &=A_{11}(\xi-\xi')+A_{21}(\eta-\eta')+A_{31}(\zeta-\zeta')+A_{41}(\nu-\nu')-i\omega_1(\nu-\nu')\\ &=A_{11}(\xi-\xi')+A_{21}(\eta-\eta')+A_{31}(\zeta-\zeta'),\ \textit{for } A_{41}=i\omega_1.\end{aligned}$$

Then

$$\int \alpha^2 d\Omega = A_{11}^2\int\frac{(\xi-\xi')^2}{R^6}d\Omega + A_{21}^2\int\frac{(\eta-\eta')^2}{R^6}d\Omega + A_{31}^2\int\frac{(\zeta-\zeta')^2}{R^6}d\Omega + 2A_{11}A_{21}\int\frac{(\xi-\xi')(\eta-\eta')}{R^6}d\Omega + \ldots$$

Now we have, since the integration extends over the space internally bounded by the sphere

$$(\xi-\xi_0)^2+(\eta-\eta_0)^2+(\zeta-\zeta_0)^2=a^2,$$

$$\int\frac{(\xi-\xi')^2}{R^6}d\Omega = \int\frac{(\eta-\eta')^2}{R^6}d\Omega = \int\frac{(\zeta-\zeta')^2}{R^6}d\Omega = \tfrac{1}{3}\int\frac{d\Omega}{R^4},$$

and from symmetry,

$$\int\frac{(\xi-\xi')(\eta-\eta')}{R^6}d\Omega=0.$$

Now we have

$$\int_{\infty}^{a}\frac{d\Omega}{R^4} = -\frac{4\pi}{a},$$

$$\therefore \int \alpha_1^2\,d\Omega = -\frac{4\pi}{3a}\left[A_{11}^2+A_{21}^2+A_{31}^2\right] = -\frac{4\pi}{3a}(1+\omega_1^2),$$

and

$$\int \alpha_1\alpha_2\,d\Omega = -\frac{4\pi}{3a}\left[A_{11}A_{21}+A_{12}A_{22}+A_{13}A_{23}\right] = -\frac{4\pi}{3a}\left[A_{14}A_{24}\right] = -\frac{4\pi}{3a}\omega_1\omega_2.$$

$$\therefore X_x = -\frac{e^2}{8\pi}\left[-\frac{4\pi}{a}(1+2\,\omega_1^2)+2.\frac{4\pi}{3a}.(1+\omega_1^2)\right] = \frac{2e^2}{3a}(\tfrac{1}{4}+\omega_1^2),$$

and $$X_y = -\frac{e^2}{4\pi}\left[-\omega_1\omega_2.\frac{4\pi}{a}+\frac{4\pi}{3a}.\omega_1\omega_2\right] = \frac{2e^2}{3a}\omega_1\omega_2.$$

Then we have similarly

$$\left.\begin{aligned}&Y_y=\frac{2e^2}{3a}(\tfrac{1}{4}+\omega_2^2), && Z_z=\frac{2e^2}{3a}(\tfrac{1}{4}+\omega_3^2),\\ &L_l=\frac{2e^2}{3a}(\tfrac{1}{4}+\omega_4^2), && X_l=\frac{2e^2}{3a}\omega_1\omega_4,\text{ \&c.}\end{aligned}\right\}\quad(5)$$

Now we have

$$F_x = -\frac{2e^2}{3a}\left[\frac{\partial}{\partial x}(\tfrac{1}{4}+\omega_1^2)+\frac{\partial}{\partial y}(\omega_1\omega_2)+\frac{\partial}{\partial z}(\omega_1\omega_3)+\frac{\delta}{\delta l}(\omega_1\omega_4)\right],$$

i.e.

$$F_x = -\frac{2e^2}{3a}\left[\left(\omega_1\frac{\partial}{\partial x}+\omega_2\frac{\partial}{\partial y}+\omega_3\frac{\partial}{\partial z}+\omega_4\frac{\partial}{\partial l}\right)\omega_1 + \omega_1\left(\frac{\partial\omega_1}{\partial x}+\frac{\partial\omega_2}{\partial y}+\frac{\partial\omega_3}{\partial z}+\frac{\partial\omega_4}{\partial l}\right)\right].$$

The second term $=0$ from the condition Div $\mathbf{a}=0$, for this gives

$$\frac{\partial}{\partial x}\left(\frac{e\omega_1}{R}\right)+\frac{\partial}{\partial y}\left(\frac{e\omega_2}{R}\right)+\frac{\partial}{\partial z}\left(\frac{e\omega_3}{R}\right)+\frac{\partial}{\partial l}\left(\frac{e\omega_4}{R}\right)=0,$$

i.e. $$\frac{1}{R}\text{ Div }\omega+(\omega_1\alpha_1+\omega_2\alpha_2+\omega_3\alpha_3+\omega_4\alpha_4)=0,$$

from which Div $w=0$, for the last term is identically zero.

The X-component of the force on the external space

$$=-\frac{2e^2}{3a}\frac{d^2x}{ds^2},\ \text{for } \frac{d}{ds}=\omega_1\frac{\partial}{\partial x}+\omega_2\frac{\partial}{\partial y}+\omega_3\frac{\partial}{\partial z}+\omega_4\frac{\partial}{\partial l}\quad(6)$$

We may interpret this force as the reaction of the electron on the external space, which is supposed for purposes of substantiation to be composed of aether. The effective force on the electron is equal and opposite to this force, and has therefore the components

$$\frac{2e^2}{3a}\frac{d^2x}{ds^2},\ \frac{2e^2}{3a}\frac{d^2y}{ds^2},\ \frac{2e^2}{3a}\frac{d^2z}{ds^2},\ \frac{2e^2}{3a}\frac{d^2l}{ds^2},$$

(N.B.—We have for small velocities $ds=cdt$ approximately,

$$\therefore \frac{2e^2}{3a}\frac{d^2x}{ds^2}=\frac{2e^2}{3ac^2}\frac{d^2x}{dt^2},\ \&c.$$

We therefore observe that the quantity $\frac{2e^2}{3ac^2}$ plays here the same part as the mass m_o. We can therefore call $\frac{2e^2}{3ac^2}$ the rest-mass of the electron, and put it equivalent to m_o.

5.

Now a few remarks on the equations (2). These were first introduced into Mathematical Physics by Maxwell about 1865. Ever since their introduction, various efforts have been made by different investigators for getting something out of them, and in certain cases they have yielded very valuable information, and led to many important results. We may cite for example, Maxwell's prediction of the existence of Radiation Pressure. The close analogy of the equations (2) with the equations of elasticity led Maxwell to propose his famous theory of "Stresses," i.e. to imagine that the electric forces are due to a distribution of the stresses (Xx, Xy . . .) in aether, which behaves in this case like an elastic solid. But this theory is fraught with many difficulties, which have been pointed out from time to time by several investigators[6]. In a paper[7] communicated to the *Phil. Mag.*, the author observed that though the forces can be well accounted for, the Energy of Electrification cannot be accounted for on Maxwell's hypothesis.

Another direction in which the equations (2) have been exploited is the subject of Electromagnetic mass of an electron. When an electron moves with a certain velocity, it creates round it an electric as well as a magnetic field. We can say with Maxwell that the energy is stored in the aether, and the electron by its motions exerts a force on every particle of aether.

If we now integrate this force over the whole space[8] exterior to the electron, the first three terms involving $\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)$ can be reduced to a surface-integral. The bounding surface is taken to be at an infinite distance, thereby the surface integrals are made to vanish. The total force on the aether thus comes out in the form

$$\frac{dM}{dl}=\frac{1}{lc}\frac{\partial M}{\partial t}.$$

Now assuming that the force exerted by the aether on the electron is equal and opposite to the force exerted by the electron on the aether, the reaction of aether on the electron becomes equivalent to $-\frac{1}{lc}\frac{\partial M}{\partial t}$. In analogy with Classical Mechanics, we can call $\left(\frac{iM}{c}\right)$ a momentum.

This is, in brief, the theory of Electromagnetic momentum as developed by Abraham, Lorentz[9], and others. We do not enter into a discussion of the rival theories of Lorentz and Abraham on the shape of the electron during motion. The electromagnetic mass is obtained from either of the relations $m_t=\frac{iM}{cv}$ and $m_l=i\frac{\partial M}{c\partial v}$, m_t and m_l denoting respectively the transverse and longitudinal masses of the electron.

But several objections can be raised to this theory of Electromagnetic momentum. In the first place, the integration is extended over the space of the observer, whereas the Principle of Relativity requires that it should be extended over the space perpendicular to the axis of motion of the electron, and external to the volume occupied by the electron. This is what I have done in the foregoing, and I believe that this is quite in keeping with Minkowski's ideas of equivalence of time and space. Secondly, the volume of integration is supposed to be bounded by a sphere at an infinite distance only, and no notice is taken of the internal boundary which must coincide with the surface of the electron. In fact, it looks as if the surface integrals had to go, because the authors wanted to get rid of them.

In the theory proposed by me, I have refrained from putting any interpretation on the quantities (X_x, X_y, . . .). Taking the theorem as it is, the total effective force on the aether has been obtained by integrating f over the whole space perpendicular to the axis of motion of the electron, the space being bounded on the inside by the surface of the electron. The "Effective force" on the electron has been taken to be equal and opposite to this force.

I may be allowed to point out here that this procedure by no means confers substantiality upon the aether. It is a fictitious creation, introduced for the sake of arriving at a result which, from its very nature, can be attempted only by indirect means.

It is remarkable that none of the quantities $\int X_x d\Omega$, & c. vanish in this case, as in the other theories. *The "Effective" force on an electron, instead of simply being the rate of change of "Momentum" becomes the sum total of the time-rate of change of the quantity* $\int\frac{\partial X_l}{\partial l}d\Omega$ *plus the space-rates of changes of the quantities* $\int\frac{\partial X_x}{\partial x}d\Omega$, $\int\frac{\partial X_y}{\partial y}d\Omega$.,. . .

[6] Maxwell, 'Electricity and Magnetism', third edition, Vol. i, chap. v, footnote p. 165.

[7] *Phil. Mag.*, March 1917.

[8] N. B. This space is the absolute space of the Pre-Relativity Period.

[9] Lorentz, 'Theory of Electrons', chap. 1, § 26 *et seq.*

These latter quantities involve "velocity" in the second order, whereas $\int \frac{\partial X_l}{\partial l}\, d\Omega$ *involves "velocity" in the first order,* so that when *the velocity is a small fraction of the velocity of light, the theorem approximates to Newton's Second Law of Motion.*

The rest-mass calculated on this basis is equivalent to $\frac{2}{3}\cdot\frac{e^2}{ac^2}$ and as such coincides with the value obtained by Sir J. J. Thomson for slow-moving electrons, and with that obtained by Lorentz and Einstein. The variation of mass with velocity is determined by the Principle of Relativity as in the theories of Lorentz and Einstein.

In conclusion, I wish to express my thanks to my friend and colleague Mr. Satyendra Nath Basu, M.Sc., for much help and useful criticism.

Calcutta University College of Science,
Physical Dept., July 10, 1917.

6. ON THE INFLUENCE OF THE FINITE VOLUME OF MOLECULES ON THE EQUATION OF STATE*

M. N. SAHA & S. N. BASU

(*Phil. Mag., Sr. VI,* **36**, 199, 1918)

It is well known that the departure of the actual behaviour of gases from the ideal state defined by the equation $p=\frac{NK\theta}{v}$ is due to two causes: (1) the finiteness of the volume of the molecules, (2) the influence of the forces of cohesion, i.e., the attractive forces amongst the molecules. van der Waals was the first to deduce an equation of state in which all these factors are taken into account; according to van der Waals, we have

$$p=\frac{NK\theta}{v-b}-\frac{a}{v^2} \tag{1}$$

where $b=4\times$ volume of the molecules, a defines the forces of cohesion.

In all subsequent modifications of this equation (Clausius, Dieterici, or D. Berthelot), the changes which have been proposed all relate to the influence of the cohesive forces; the part of the argument dealing with the finiteness of molecular volumes is generally left untouched.

But it has been found that the results of experiments do not agree with the predictions of theory if we regard a and b as absolute constants. Accordingly it has been proposed to regard both a and b as functions of volume and temperature.[1]

But before proceeding to these considerations, it is necessary to scrutinize whether the influence of finite molecular volumes is properly represented by the term b. From theoretical considerations, the conclusion has been reached that this is not the case. The argument is as follows: According to Boltzmann's theory,

the entropy $S=K\log W+C$,

where K=Boltzmann's gas constant, W=probability of the state. Let us now calculate the probability that a number of N molecules originally confined within the volume V_0 and possessing finite volumes, shall be contained in a volume V. Neglecting the influence of internal forces, the probability for the first molecule is $\frac{V}{V_0}$, for the second molecule the probability is $\frac{V-\beta}{V_0-\beta}$, where $\beta=8\times$ volume of a single molecule, for when the first molecule is in position, the space enclosed by a concentric sphere of double the radius of the molecule will not be available for the second molecule. The available space is therefore $V-\beta$, whence the probability is $\frac{V-\beta}{V_0-\beta}$. Introducing similar considerations for the rest of the molecules, we have

$$W=\frac{V}{V_0}\cdot\frac{V-\beta}{V_0-\beta}\cdot\frac{V-2\beta}{V_0-2\beta}\cdots\frac{V-\overline{N-1}\beta}{V_0-\overline{N-1}\beta} \tag{2}$$

We are, of course, neglecting those cases in which partial overlapping of the regions occupied by two or more molecules occurs; for the number of such cases can at best be a small fraction of the total number. Even cases of actual association do not include these, for in that case, two discrete molecules become merged into one, without their outer surfaces being actually in contact.

From the relations $S=K\log W+C$

and $\left(\frac{\partial S}{\partial V}\right)_u=\frac{p}{\theta}$

we can easily verify that

$$p=-\frac{K\theta}{\beta}\log\frac{V-n\beta}{V}$$
$$=-\frac{R\theta}{2b}\log\frac{V-2b}{V}\,(R=NK) \tag{3}$$

*Communicated by the Authors.

[1]Compare van der Waals, *Proc. Amst.*, 1916; Van Laar, *Proc. Amst.*, vol. xvi. p. 44.