4. ON THE PRESSURE OF LIGHT

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The object of the present paper is to describe a simple apparatus by means of which the pressure of light can be easily demonstrated, and qualitatively measured with the entire elimination of all sorts of disturbing effects. The materials required are not difficult to procure, and are readily available in all well-equipped laboratories.

We wish first to give a short history of the subject and a short sketch of the theory[1]. As early as the seventeenth century Kepler supposed that light exerted a pressure on surfaces on which it is incident. The hypothesis was called into being for explaining the tails of comets.

With the rise of Newton's corpuscular theory of light, the pressure no longer remained a guess, but could be deduced from that theory. An elaborate series of experiments for detecting the pressure were instituted by De Mairan (1754), and later on by Du Fay (1756), but the results were entirely negative. Later on, the failure of these experiments were used as arguments against the validity of the corpuscular theory of light.

But interest in the subject was again revived when Maxwell[2], in the year 1873, predicted that even on the basis of the electro-magnetic theory of light, radiant energy should exert a pressure on a surface on which it is incident. But the amount of pressure is extremely small. It can be shown that if light consists of unidirectional rays, the pressure amounts to $\frac{1}{c}$ (amount of radiant energy falling on unit surface per unit of time, measured in absolute units), where c is the velocity of light, and the surface is a perfectly absorbing one, e.g. a surface coated with lamp-black.

If the surface on which the light is incident be perfectly reflecting, the pressure is just double. But if, on the other hand, the surface be transparent (e.g. glass), there will be no pressure at all, or more accurately a very small amount of pressure depending on the small amount of reflection from the glass surface.

The occurrence of the term c in the denominator makes the pressure extremely small. Let us take for example the pressure exerted by solar light. The amount of energy which is delivered by the sun on unit surface placed normally to the rays of the earth is equivalent to 2·4 calories per minute. The pressure therefore

$$=\frac{2\cdot4\times4\cdot2\times10^{7}}{3\times10^{10}\times60\times981}\ \text{gms. weight}=\cdot56\times10^{-7}\ \text{gms.weight}.$$

By using the arc, or a very high candle power filament lamp (1500 wt/$\frac{1}{2}$ wt. for example), and by concentrating the light by means of a lens of large aperture, the pressure can be increased to about 100 times. But still it is extremely small.

It was for demonstrating the pressure of light that Crookes[3] was led to invent his famous "radiometer". As is well known, this consists of a delicate cross of glass or mica vanes suspended on a pivot and enclosed within a glass cylinder from which air can be pumped off at will. The alternate faces of the vanes are covered with lamp-black. When light falls on the vanes it begins to rotate rapidly about the axis.

Crookes was inclined to explain this motion as being due to the pressure of radiant energy, but Zöllner[4] showed that the effect observed was rather spurious, and exceeded theoretical pressure by at least 10^3 times. He showed that the effect was really due to the unequal heating of the two sides of the vanes.

Zöllner tried to observe the effect by another arrangement. Two thin discs of silvered or blackened glass, or metal, were suspended at the ends of the horizontal arm of a thin cross of glass-rods and the whole was suspended by means of glass fibres within a closed vessel, from which air can be pumped out at will. A galvanometer mirror is attached to the vertical part, with its plane at right angles to the plane of the vanes. But with light incident on the vanes, the deflection observed was very irregular, and sometimes was completely in the wrong direction.

But in spite of repeated failures to detect the pressure of radiation, theoretical investigation had, in the meantime, been advanced so far that it was not possible to deny its existence.

We have seen that the pressure of light was deduced by Maxwell from the electromagnetic theory of light, by using an argument involving the assumption of pressures and tensions across and along tubes of force. But Bartoli[5] showed in 1877 that the pressure could also be deduced by means of thermodynamic reasoning involving only the two laws of thermodynamics, and was in amount just the

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[1] For the historical part, see Lebedew, *Ann-d. Phys.*, Bd. 6, p. 433; and Nichols and Hull, *Phys. Rev.*, 1903.

[2] Maxwell, Electricity and Magnetism, Vol. II, p. 792.

[3] *Phil. Trans.*, 1874, Vols. 164, p. 501.

[4] *Pogg. Ann Bd.*, 160, p. 154, 1877 (suggested by Maxwell).

[5] Bartoli, *Nuovo Cimento*, 15, p. 195, 1883.

same as is obtained from Maxwell's theory. Bartoli's argument being based on the surer basis of thermodynamics, seemed to carry conviction in all quarters about the real existence of the pressure.

The long-expected pressure was at last observed by Lebedew, and almost simultaneously by Nichols and Hull in 1901, by different modifications of Zöllner's unsuccessful experiment.

Lebedew's method was to replace the rather thick glass vanes by means of very thin platinum foils (diameter 5 mm., thickness ·02 mm.) whereby any difference of temperature on the two sides is instantly equalised. The radiometer action is directly proportional to the difference of pressure on the two sides, and the pressure of gas within the vessel. Lebedew reduced the pressure to about 1/20000th of a mm. and was almost able to eliminate the radiometer action, and verify the pressure qualitatively to about 20% of the theoretical pressure.

The early experiments of Nichols and Hull were undertaken in order to investigate the different disturbing effects in the apparatus of Zöllner. They found that the total disturbing effect is the resultant of the following:—

(*i*) the radiometer action—due to the unequal heating of the two sides of the vane;

(*ii*) convection currents—due to the rush of air towards the parts warmed by the passage of the pencil of rays;

(*iii*) a rocket action—due to the escape of particles of gas from the surface of the vanes when these are heated by the incident light.

By a series of elaborate investigations extending over three years, Nichols and Hull were able to get rid of these effects. They found that the convection effect could be reduced by making the vanes exactly vertical, for then the flow of air becomes tangential to their surface. The rocket action, and the radiometer action were found to balance at a pressure of 16·5 mm., and deflections were therefore observed with this pressure in the vessel. The vanes were of thin glass with one face silvered; for further information on the point reference should be made to the original paper.

Finally, Hull[6] evolved out an arrangement by means of which the disturbing effects could be entirely eliminated. The silvered side of a thin cover-glass was placed in contact with the blackened side of a similar glass and the whole was enclosed by means of two other thin glasses, as shown in the figure. Two such cells were mounted upon the opposite ends of the torsion arm which was suspended by means of a thin quartz fibre within a glass cylinder. When light falls on the vanes, the two sides are of course unequally heated. But as the air on the two sides is enclosed within a glass cylinder, it forms one single system with the glass vessel—action and reaction being equal, the radiometer action is entirely eliminated.

Fig. 1

We have found the extra glass cell to be redundant. The silvered sides of two thin cover-glasses were put one upon the other and connected to each other by means of a trace of Canada balsam on the fringes. Similarly, we prepared a lampblacked surface. We have thus in these vanes very thin films of totally reflecting and totally absorbing material enclosed within equal thicknesses of glass on either side. When light previously filtered of all rays capable of heating glass, is allowed to fall on one of the vanes, say the silvered one, the glass surface is not at all heated by the passage of the rays, which have been previously passed through sufficiently thick glass lenses. The two sides of the film are instantly raised to the same temperature (because they are extremely thin and there being equal thicknesses of glass on the two sides, they are equally heated by conduction). Thus the radiometer action is entirely eliminated.

It will be thus seen that in our arrangement we have combined the arrangements of Lebedew as well as Hull's method, without the additional encumbrance of extra glass cells.

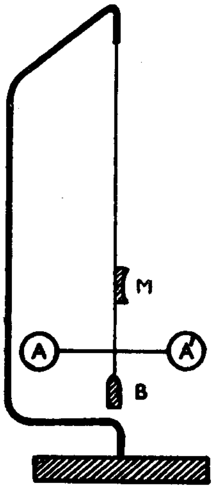

Fig. 2

Description of the Apparatus

The vanes were suspended on the opposite arms of the torsion balance: *vide* fig. 2. (M) is a galvanometer mirror placed at right angles to the plane of the vanes, with a small piece of steel on its back. (B) is a small brass weight for steadying the balance. The whole is suspended by means of a glass fibre and enclosed within a bell-jar which is connected to a pump and a manometer. The deflection is observed from the excursion of a spot of light reflected from (M) in the usual lamp and scale arrangement. The dimensions are

Diameter of the cover glasses = 1·8 cm.
Thickness of the cover glasses = ·083 mm.
Weight of the silvered vane = ·105 gm.
Weight of the lampblacked vane = ·128 gm.
Length of the arm = 2 cm.
Weight of B = ·5 gm.

The pressure within the bell-jar is reduced to about 1 to 2 cms. of mercury. It is extremely important that the joints

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[6] Hull, *Phys. Rev.*, May 1905.

should be all air-tight, for the slightest leakage of air may produce disastrous effects. After pumping out we allowed the apparatus to stand for 3 days in order to be sure that it was quite air-tight. The vanes should be placed symmetrically just about the centre of the glass vessel, otherwise currents of air which are set up in the vessel by the passage of rays and turned off by the sides of the vessel may produce disturbing effects. These effects become smaller, the smaller the pressure inside the vessel.

Theory of the Apparatus:

The equation of motion of the vanes is given by—

$$I\frac{d^2\theta}{dt^2}+k\frac{d\theta}{dt}+\mu\theta=L \qquad \text{(i)}$$

where (I) moment of inertia of the system about the fibre, k viscosity factor, μ is the torsional coefficient, θ=angle of rotation, L moment of the force of pressure about the axis of rotation (i.e. the fibre).

The solution is $$\left(\theta-\frac{L}{\mu}\right)=Ae^{-\frac{k}{2I}t}\cos(nt+\alpha) \qquad \text{(ii)}$$

where $$n^2=\frac{\mu}{I}-\frac{k^2}{4I^2} \qquad \text{(iii)}.$$

After a sufficiently large time the deflection should become steady if the disturbing causes are entirely absent. Let α denote this steady deflection.

Now $L=pl$, where p=total pressure (or thrust) on the surface and l=distance of the centre of the disc from the axis of rotation. The light should be concentrated on the centre of the disc. Let α be the steady deflection. Then

$$p=\frac{\mu\alpha}{l}.$$

The constant μ is obtained from observations of the free period of oscillation of the system.

From (ii) we see that $\frac{\mu}{I}=n^2+\frac{k^2}{4I^2}$.

Now $n=\frac{2\pi}{T}$, and $\frac{k^2}{4I^2}=\left(\frac{\beta}{T}\right)^2$, where β=logarithmic decrement of the amplitude.

$$\therefore\frac{\mu}{I}=\left(\frac{2\pi}{T}\right)^2+\left(\frac{\beta}{T}\right)^2=\frac{1}{T^2}(4\pi^2+\beta^2) \qquad \text{(iv)}$$

Now I can be easily calculated from the weight and the dimensions of the system. μ can therefore be easily calculated from formula (iv).

In our experiment $l=2\cdot 65$ c.m. and $\alpha=6\cdot 27\times 10^{-5}$ so that a deflection of (1 mm.) at a distance of 1 metre corresponded to a total pressure of

$$\frac{6\cdot 27\times 10^{-5}}{2\cdot 65}=2\cdot 36\times 10^{-5}\text{ dynes.}$$

The time period was 32 seconds and the logarithmic decrement was $\beta=\cdot 310$, and $I=1\cdot 67$ units.

MEASUREMENT OF ENERGY.

Owing to lack of means at our disposal the amount of energy falling upon the surface could not be properly measured. Lebedew allowed the light to fall on a copper calorimeter placed in the same position as the vanes, and the amount of energy absorbed was obtained by noting the rise in temperature of the calorimeter within a given period of time.

Nichols and Hull's method was more ingenious. A thin disc of silver of the same size as the vane was coated with lampblack. Two holes were bored on the sides through which a copper-constantan couple passed. The other end of the couple passed through a sensitive galvanometer. This apparatus was previously standardised by putting it in different baths. The light was allowed to fall on the disc for some time and the rise in temperature was obtained from the throw of the galvanometer.

The source of light in Lebedew and Hull's experiment was an arc which as is well known is very unsteady. In our early experiments we used the arcs but in the latest experiment the source of light was a (3000 c.p.) Tungsten filament lamp supplied by Messrs. Westinghouse & Co. The light from this source is very steady. The lamp was placed in a horizontal position (i.e. with its filament in a vertical circle) at a distance of 50 to 70 cms. from the diaphragm which contained a short focus lens of 6·5 cms. apperture. By adjusting the lens the filament was completely focussed on the vanes. An upper limit to the amount of energy falling on the vane per second can be thus obtained. By means of an ammeter we found that the lamp consumed a current 6·6 amps. under a pressure of 220 volts. The amount of energy passing through the lens and focussed on the vanes is therefore given by

$$\frac{220\times 6\cdot 6\times 10^7}{4\pi(d)^2}\pi(3\cdot 25)^2\text{ ergs per sec.}$$

The whole pressure on the silvered surface is therefore

$$\frac{E}{c}(1-\epsilon)(1+\rho)$$

where c=velocity of light and ϵ=fraction of energy absorbed by and reflected from glass surfaces (lens and containing vessels) and ρ=fraction of energy reflected from the silvered face.

RESULTS OF OBSERVATIONS.

In our preliminary blank experiment with the arc, we found that for the period for which the arc remains steady, the deflection remains quite steady and follows very faithfully the fluctuations of the arc. When the positive pole was focussed the deflection observed was generally 3 to 4 times the deflection for the negative pole. When all the precautions above mentioned were taken, the deflection was found to be always in the right direction. When the filament lamp was used as the source of light, all irregularities due to the variation of the source of light vanished. As soon as light is struck, the spot of light slowly creeps up towards the new position of equilibrium about which it oscillates in accordance with the equation (i).

Ultimately the oscillation dies away and the spot becomes quite steady, which could be maintained for 15 minutes (we did not try to keep the spot steady for a greater length of time because the tungsten filaments, being kept in a horizontal position, are gradually deformed on account of their plasticity at the high temperature within the lamp).

In one set of experiments one of the vanes was silvered while the other consisted of two clear pieces of microscopic cover glass. We found that when light was allowed to fall on the clear glass surface there was practically no deflection. In another set of experiments one of the vanes was silvered and the other was lampblacked. It was found that generally if the source of light was not too intense, the deflection of the black surface was approximately one half of that of the silvered one. If the source of light was very intense so much heat was absorbed that the junctions (which were all of shellac) melted off. Quantitative experiments were therefore impossible with that surface.

One of the results of our quantitative experiments is given below:—

Mean deflection (mean of several experiments) =28·5 Divns.
Distance of the scale from the mirror =100 cm
Distance "d" of the plane of the filament from the diaphragm =73 cm

Therefore the upper limit of the total theoretical pressure (without allowing for absorption or reflexion) is equal to

$$2\times\frac{6{\cdot}6\times 220\times 10^{7}\times(3{\cdot}25)^{2}}{4\times 73^{2}\times 3\times 10^{10}}=4{\cdot}8\times 10^{-4}\text{ dynes}\qquad\text{(A).}$$

The pressure calculated from deflections is equal to

$$2{\cdot}3\times 10^{-5}\times 14{\cdot}25=3{\cdot}33\times 10^{-4}\text{ dynes}\qquad\text{(B).}$$

The observed pressure is about 70 per cent of the expression (A), which is the pressure calculated on the supposition that the whole amount of energy given out by the filament is freely transmitted by the various glass media, and is totally reflected by the silvered surface. As a matter of fact, none of these assumptions is correct. If T is the fraction of total energy transmitted by thick glass, and ρ be the reflecting power of a silver glass-surface the actual pressure should be

$$P_0\frac{T}{2}(1+\rho)\ (1-\epsilon)$$

where P_0 is the quantity (A).

According to the experiments of Rubens and Hagen[7] $\rho=90{\cdot}5\%$ unfortunately no data is available for the transmission coefficient, but on account of the preponderance of rays of short wave length in the spectrum of the light from a tungsten filament, it cannot be less than 80%.

Considering these facts, we are probably justified in asserting that the agreement between observed and theoretical values is at least qualitatively quite good. On a future occasion we hope to return to the problem of a rigorous quantitative determination of total incident energy.

In conclusion, we beg to record our best thanks to Prof. C. V. Raman, and the teaching staff of the University College of Science, for the interest they have taken in the work; and to Mr. N. Basu, B.Sc., for much useful help.

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[7] Obtained by extrapolation from the data of Rubens and Hagen on the supposition that the maximum emission of energy from a tungsten filament is at 1 μ [Kohlrausch, Praktische Physik, Tabellen].

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5. ON THE DYNAMICS OF THE ELECTRON*

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Mass as a fundamental physical concept has been introduced into Physics by Newton's Second Law of Motion, which may be said to form the corner-stone of classical Mechanics. But in spite of its splendid success, physicists have always encountered some difficulty in realising mass as a fundamental physical concept in the same sense as the concepts of time and space. The fundamental object of mechanics is to provide a scaffolding by means of which the motion of material bodies can be surveyed and followed, when these are subjected to various disturbing influences. Some hypothesis must be introduced for taking into account the influences of these disturbing agencies. The question is: "Are Newton's Second Law of Motion and the ideas underlying it quite sufficient for all possible cases of motion, or are we to search for some more general principle ?" Some physicists are in fact in favour of introducing Energy as a more fundamental physical concept than Mass, thereby basing the Science of Mechanics on various Energy-theorems.

So long as we hold to the principle of invariability of

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*Communicated by Prof. A. W. Porter, F.R.S.