EXPERIMENTAL CONFIRMATION

These results have been experimentally confirmed. Berkner states that at Huancayo: The ordinary ray is polarized with its electric vector along the magnetic north-south. Table (1) shows that
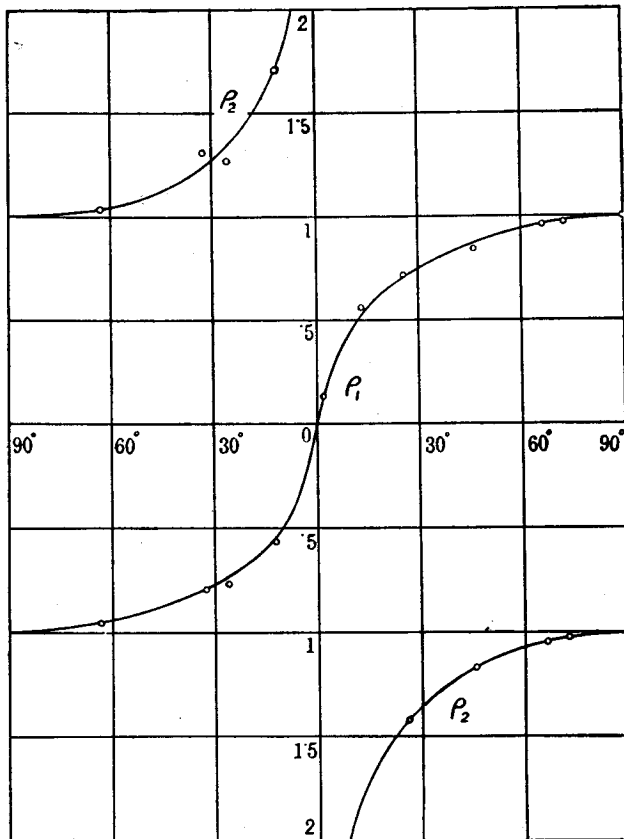
$$E_x\text{-axis}: E_y\text{-axis} = 1 : \cdot 129$$

i.e. the electric vector is mainly along the x-axis, i.e. magnetic north-south.

The extraordinary ray is polarized with its electric vector along the magnetic east-west. Table (1) shows that

$$E_x\text{-axis}: E_y\text{-axis} = 1 : 7\cdot 643.$$

The variation of polarization for the o- and x-waves with latitude are shown in fig. 2.

CONCLUSION

It is shown that if the complex refractive index be regarded as constant we get the same conditions for reflection and polarization of the radio-waves for vertical propagation as was obtained by Appleton. But the refractive indices vary with height, hence the treatment given here should be replaced by a wave-treatment. A simple case of wave-treatment has already been published in paper 2.

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Fig. 2

70. ON THE IONIZATION OF THE UPPER ATMOSPHERE

M. N. SAHA AND R. N. RAI

1. INTRODUCTION

There are at present two works attempting a theoretical explanation of the ionization of the Upper Atmosphere. First, the work of A. Pannekoek (1926), which is thermodynamical, and is based upon Saha's theory of thermal ionization of atoms as extended by Milne (1924) and Woltjer (1925) to material systems traversed by radiation from an external body at a higher temperature. The second method is that of S. Chapman (1931 *a, b*) who considers the ionization produced by the absorption of a monochromatic beam of light in an atmosphere in which the density is assumed to vary exponentially. Prof. Chapman in a Bakerian lecture in 1931 gave accounts of both theories side by side, but he does not appear to have tried to demonstrate the connection between the two methods of calculating the ionization of the upper atmosphere. It will be shown presently that the two theories are not

really different; when Chapman's theory is properly developed and extended, it leads to the same result as that of Pannekoek.

2. PANNEKOEK'S WORK

It is necessary to start with a critical description of the main results of these two theories. Pannekoek (1926) shows that the number of electrons produced by the ionization of any one of the constituents in the earth's atmosphere can be obtained from the formula

$$K = \frac{n_+ n_e}{\mathcal{N}} = \left(\frac{n_+ n_e}{\mathcal{N}}\right)_0 \frac{\int_{\nu_0}^{\infty} \psi(\nu).I(\nu).d\nu}{\int_{\nu_0}^{\infty} \psi(\nu).e^{-\frac{h\nu}{kT}}\left\{\frac{8\pi h\nu^3}{c^2}+I(\nu)\right\}.d\nu},$$

$$\cdots \quad (1)$$

where

$\left(\frac{n_+ n_e}{\mathcal{N}}\right)_0 = K_o =$ Reaction isochore under equilibrium conditions,

$\mathcal{N}=$ the number of atoms per unit volume,

$n_+=$ the number of ions per unit volume,

$n_e=$ the number of electrons per unit volume,

$I(\nu)=$ the intensity of radiation of frequency ν.

$\psi(\nu)$ denotes the *probability of ionization* by light of unit intensity. $\psi(\nu)$ is connected with the atomic absorption coefficient $\tau(\nu)$ by the relation

$$\psi(\nu) = \tau(\nu)/h\nu.$$

There are two things to be noticed about this formula. First, that ionization starts at a frequency ν_0 and is produced by radiation of higher frequency just as we know from laboratory experiments. Secondly that the values of the density of electrons or of positive ions which we obtain from the above formula are *equilibrium values*, i.e. they are the values of concentration when a condition of equilibrium has been established between the rate of ionization due to solar radiation and the rate of disappearance due to recombination or any other process.

3. CHAPMAN'S WORK

Chapman (1931*a*) shows, following an earlier work by Lenard (1911), that a monochromatic beam of radiation, of intensity $I(\nu)_0$ just outside the earth's atmosphere, is reduced in its passage through the earth's atmosphere, at a slanting angle χ, to the intensity $I(\nu)$ given by the formula

$$I(\nu) = I(\nu)_0 \operatorname{Exp}\left\{-A(\nu).\rho_0.H.\sec\chi.e^{-\frac{z}{H}}\right\}. \quad (2)$$

Here $A(\nu)$ is the mass absorption coefficient, and $H=kT/Mg$ is the height of the homogeneous atmosphere. The temperature is assumed to be constant.

Then we have

$$\frac{dI(\nu)}{dz} = I(\nu)_0.A(\nu).\rho_0.\sec\chi.\operatorname{Exp}\left\{-\frac{z}{H}-A(\nu).\rho_0.H.\sec\chi.e^{-\frac{z}{H}}\right\}.$$

It is next supposed that the number of electrons or of positive ions produced is equal to $\beta\dfrac{dI(\nu)}{dz.\sec\chi}$. Hence the number of electrons produced per second per unit volume is given by

$$q(\nu) = S.A(\nu).I(\nu)_0\,\rho_0\,\operatorname{Exp}\left\{-\frac{z}{H}-A(\nu).\rho_0 H.\sec\chi.e^{-\frac{z}{H}}\right\}.\,(3)$$

It is easy to see that $\beta=\dfrac{1}{h\nu}$, but Chapman does not attempt to define β and plots $q(\nu)$ as a function of z and thus gets a curve of variation of density of ions in the earth's atmosphere with height. His theory explains to some extent the variation of electron density of the ionized layers in the course of the day as observed in the radio experiments. Further he shows that the values of maximum electron density in winter and summer are related as

$$\frac{\mathcal{N}_e^W}{\mathcal{N}_e^S} = \left\{\frac{\sin(\theta-\delta)}{\sin(\theta+\delta)}\right\}^{\frac{1}{2}}, \quad (4)$$

where δ is the declination of the sun and θ the colatitude of a place. This relation has been verified approximately by Appleton, Naismith and others (1935).

It should however be mentioned that it is not possible to arrive at an *absolute value* of $q(\nu)$ from Chapman's formula as the constant β is not precisely defined. Secondly, Chapman's theory holds only for *monochromatic radiation*, but if we suppose that the sun radiates like a black body, ionization will be produced by continuous radiation in the way supposed by Pannekoek. The values which Chapman obtains are not equilibrium values n_e of ionization density, but denote the *number of ions* produced per second by the radiation. But what we measure in radio experiments is the *equilibrium value*. For unicomponent systems, it can be shown (Appleton, 1938) that $n_e=(q/\kappa)^{\frac{1}{2}}$, but it is hardly correct to say that in the ionosphere we have to deal with a unicomponent system.

To show the connection between the two theories, let us first find out the relation between equilibrium value, the rate of production of ions and the rate of recombination. Let us take the simplified case of $\mathcal{N}_2$. As a result of reaction with sunlight, we can expect that the $\mathcal{N}_2$ molecule will give rise to the following products:

$$\mathcal{N}_2,\ \mathcal{N}_2{}^+,\ \mathcal{N}_2{}^-,\ \mathcal{N},\ \mathcal{N}^+,\ \mathcal{N}^-,\ e.$$
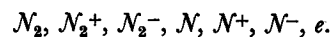
Of these we can exclude the probability of the existence of $\mathcal{N}_2{}^-$ and $\mathcal{N}^-$ because, according to a large number of experiments with the mass-spectrograph, the existence of

such ions is doubtful (see for example Tüxen, 1936). We can also neglect N and N^+ since, on account of the high dissociation potential of nitrogen, it is rather improbable that there is an appreciable number of free nitrogen atoms and atom-ions present in the atmosphere. We are, therefore, left-with only N_2, N_2^+ and electrons. Even if some of these assumptions are proved to be incorrect, it does not interfere with the general line of argument followed in the present paper, as we propose here to deal with an idealized state only. Further, let us suppose that the ionization of N_2 proceeds in the same way as for the nitrogen atom, i.e. the ionization suddenly starts with a sharp maximum at the frequency corresponding to the ionization potential of N_2 and then falls off approximately as $1/\nu^3$ for higher frequencies, although it will be shown in another paper that these assumptions do not strictly hold for the ionization of the molecule.

Let us suppose that

$$N,\ n_+,\ n_e$$

are respectively the numbers per c.c. of neutral nitrogen molecules, ionized nitrogen molecules and electrons. We of course for the present assume that $n_+ = n_e$ (unicomponent system). We have then

$$\left.\begin{array}{l} \dfrac{dN}{dt} = -q + \alpha n_+ n_e = -q + \alpha n_e^2 \\[2mm] \dfrac{dn_e}{dt} = q - \alpha n_+ n_e = q - \alpha n_e^2 \end{array}\right\}, \qquad (5)$$

where q is the number of electrons or ions produced by sunlight per c.c. and α is the *recombination coefficient*.

When equilibrium has been established, we should have $n_e = (q/\alpha)^{\frac12}$. In general, however, electrons arise not merely from the ionization of N_2, but also from O_2, O and probably N, hence the electron-concentration is to be regarded as an independent component. We shall not deal with this matter in this paper, but confine our attention only to calculation of the quantities q and α. We can easily obtain q from an extension of Chapman's method to continuous radiation. The number of ions produced by the absorption of radiation of intensity $I(\nu)$ is given by

$$q(\nu)d\nu = \frac{I(\nu).A(\nu).\rho.d\nu}{h\nu} = \frac{N.I(\nu).\tau(\nu).d\nu}{h\nu} = N\,I(\nu).\psi(\nu).d\nu, \quad (6)$$

because $I(\nu).d\nu.A(\nu)\rho/h\nu$ is the number of quanta absorbed and each quantum produces one ion. To prove the relations stated here, we observe that

$A(\nu)$ = mass absorption coefficient = $\tau(\nu)/M$, where $\tau(\nu)$ is the absorption per atom, and M is the mass of the atom.

It will be seen that eq. (6) differs from eq. (3) in having β replaced by $\dfrac{1}{h\nu}$. Further we have as yet made no suggestion regarding the variation of N or $I(\nu)$ with height.

We have, therefore, for ionization by continuous radiation

$$q = \int_{\nu_0}^{\infty} q(\nu).d\nu = N \int_{\nu_0}^{\infty} I(\nu).\psi(\nu).d\nu. \qquad (6')$$

According to Milne (1924), the recombination coefficient is given by

$$\alpha = \int_{\nu_0}^{\infty} 8\pi^2 \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac12 \frac{mv^2}{kT}} \{F(v) + I(\nu).G(v)\}v^3.dv.$$

By comparing the results obtained in the case of thermodynamical equilibrium and substituting $\frac12 mv^2 = h(\nu - \nu_0)$ this transforms into

$$\alpha = e^{\frac{h\nu_0}{kT}} \left(\frac{h^2}{2\pi mkT}\right)^{3/2} \int_{\nu_0}^{\infty} \psi(\nu)\left\{\frac{8\pi h\nu^3}{c^2} + I(\nu)\right\} e^{-\frac{h\nu}{kT}}.d\nu. \qquad (7)$$

Equating the value of $\alpha n_+ n_e$ with q, we have

$$K = K_0 \frac{\displaystyle\int_{\nu_0}^{\infty} \psi(\nu).I(\nu).d\nu}{\displaystyle\int_{\nu_0}^{\infty} \psi(\nu).e^{-\frac{h\nu}{kT}}\left\{\frac{8\pi h\nu^3}{c^2} + I(\nu)\right\} d\nu}, \qquad (8)$$

because

$$K_0 = \left(\frac{2\pi mkT}{h^2}\right)^{-\frac32} e^{-\frac{h\nu_0}{kT}}$$

is the value of the reaction-isochore, when radiation is at the same temperature with matter.

This is the original method of deduction by Milne of the equation of reaction-isochore under the conditions stated above. It is well known that assuming $\psi(\nu)$ to be given by C/ν^3 and taking $I(\nu)$ in the denominator to be negligible in comparison with $8\pi h\nu^3/c^2$, we arrive at Pannekoek's result, which will be discussed more in detail in section 5. We shall first of all calculate the rate of production of electrons, and the recombination coefficient according to formulae (6') and (8). For this purpose it is necessary to know how $\tau(\nu)$ varies with ν. This problem is discussed in the next section.

4. The Law of Photo-electric Absorption

It is generally assumed (Pannekoek, 1926) that $\tau(\nu)$ varies according to the law first given by Kramers,

$$\tau(\nu) = \frac{16}{3\sqrt{3}} \frac{\pi^2 e^6}{c\,h^3} Z^2 . \frac{\nu_0}{\nu^3}. \qquad (9)$$

Here 'Z' is the effective charge on the nucleus, ν_0 is the threshold value of absorption frequency. If we wish to apply this formula to photo-ionization of atoms and to molecules like N_2 and O_2, we have to introduce some assumption regarding the effective value of 'Z', which is generally

understood to be the nuclear charge minus the 'screening constant' due to the effect of external electrons. Pannekoek (1926) and Chapman (1931) have introduced values for O and N-atoms, and N_2 and O_2-molecules for which their papers may be consulted.

It is, however, doubtful whether Kramers' formula (9) which was first deduced on an older form of the quantum theory to account for X-ray absorption can at all be applicable to optical absorption. Rosseland (1936) comments 'One sometimes has the feeling that the applicability of the formula has been strained beyond the breaking point.'

The best course would be to take $\tau(\nu)$ values obtained from actual experiments. But as emphasized by Saha (1937), accurate experiments have not yet been performed for O_2 and N_2; and for O and N, probably the experiments would be extremely difficult. Under such circumstances, the best course appears to be to fall back upon wave-mechanical considerations.

For the H-atom, it has been shown by several authors [(for a comprehensive account, see Bethe (1932)], that $\tau(\nu)$ is given by

$$\tau(\nu)=\frac{2^8}{3}\cdot\frac{\pi\ e^2}{mc}\cdot Z^2\cdot\frac{\nu_0^3}{\nu^4}F\left(\sqrt{\frac{\nu_0}{\nu-\nu_0}}\right), \qquad (10)$$

where $F(x)=\mathrm{Exp}\ \{-4x.\ \cot^{-1}x\}/(1-e^{-2\pi x})$, and for moderate values of $\nu-\nu_0$, we have approximately (ν up to $3\nu_0$).

$$F(\nu)=\frac{\epsilon^{-4}}{3}\left\{\frac{4\nu}{\nu_0}-1\right\}, \qquad (10')$$

where ϵ is the base of natural logarithms.

We have from (10), and for $Z=1$

$\tau_0=$limiting value of $\tau(\nu)$ at $\nu=\nu_0$

$$=\frac{2^8}{3}\cdot\frac{\pi e^2}{mc}\cdot\frac{1}{\nu_0}\ \epsilon^{-4}=\frac{2^7}{3}\frac{\epsilon^{-4}.h^3}{\pi e^2 m^2 c}=1.27\times10^{-17}\ cm^2,\ .. \quad (11)$$

and $\qquad \tau(\nu)=\frac{\tau_0}{3}\left(\frac{4}{x^3}-\frac{1}{x^4}\right)$, where $x=\nu/\nu_0$. $\qquad (12)$

We have $\qquad \displaystyle\int_{\nu_0}^{\infty}\tau(\nu)d\nu=\frac{5}{9}\tau_0\nu_0. \qquad (13)$

It will be interesting to compare the value of τ_0 obtained from (11) with that obtained from Kramers' formula. For the H-atom, we have $Z=1$, and

$$\tau_0(\text{Kramers})=\frac{4}{3\sqrt{3\pi^2}}\frac{h^3}{e^2 m^2 c}=3.98\times10^{-18}\ cm^2, \quad (14)$$

$$\tau_0(\text{Wave-mechanics})=\frac{2^5\sqrt{3\pi}}{\epsilon^4}=3.19\ \tau_0(\text{Kramers}). \quad (15)$$

Kramers' value of τ_0 for the H-atom is therefore 3·19 times smaller than the wave mechanical value.

It should, however, be pointed out that the approximation (12) is only rough. This can be seen from a calculation of the oscillator strength for the continuous spectrum of hydrogen. According to a general theorem

$$f_c=\frac{g_i}{g_j}\cdot\frac{mc}{\pi e^2}\int_{\nu_0}^{\infty}\tau(\nu).\ d\nu=\frac{g_i}{g_j}\cdot\frac{Z^2.2^8}{3}\ \epsilon^{-4}\cdot\frac{5}{9}=.289.Z^2. \quad (16)$$

Here

$g_i=$weight of the normal state of the H-atom; for H, it is $1s$, $g_i=2$

$g_j=$weight of the final state of the H-atom; for H, it is ∞p, $g_j=6$.

The actual value of f_c for H is, however, ·43. This discrepancy is due to our use of the approximation (12). The above formula for f_c holds only for the hydrogen-like atoms. But in the Upper Atmosphere, it is the following processes which give rise to electrons:

$$O_2\rightarrow O_2{}^+,\ \ N_2\rightarrow N_2{}^+,\ \ O\rightarrow O^+,\ \ N\rightarrow N^+.$$
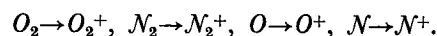
For calculating the ionization according to each of these processes, we require a knowledge of the corresponding $\tau(\nu)/\nu$-curves. It can be assumed that for all atoms, the $\tau(\nu)$ curve is given by a formula of the type (10) or its approximate form (10') which we use here where Z is to be adjusted. In fact, Vinti (1933) has actually deduced theoretically such an expression for the continuous absorption by helium.

Molecular Ionization

But a little reflection shows that the same considerations cannot apply to molecules. For here ionization is attended usually with a change in the nuclear distance, e.g. in H_2, the nuclear distance is ·75 × 10⁻⁸ cm. while for $H_2{}^+$, it is 1·071 × 10⁻⁸ cm. Hence according to the Franck-Condon principle, the value of $\tau(\nu)$ at the threshold potential is likely to be very small as the nuclear distances are very different. It will gradually rise to a maximum and then fall off. Probably the curve would be similar to that experimentally found by R. Ladenburg (1933) for the photo-dissociation of the O_2-molecule by light of wavelength $<\lambda$ 1750A; for Ladenburg's case, a theoretical expression has been found by Stuekelberg (1932). We shall not therefore, in this paper, discuss molecular photo-ionization at all.

Ionization of O and N-atoms

This leaves us only with the ionization of the O-atom and the N-atom. It has been surmised by several workers that the O_2-molecule, in the course of the daytime, is completely broken up into atoms. Recently a theory of photo-dissociation of molecules has been worked out by Dr. R. C. Majumdar (1938) at the suggestion of the senior author, and making use of Ladenburg's figures for the

variation of the absorption-coefficient with ν, he has shown that O_2 in the daytime is completely borken up into atoms at a height of 150 kms.

The process $O + h\nu = O^+ + e$ therefore appears to be of great practical value in the production of electrons in the Upper Atmosphere in the daytime.

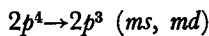We shall now consider how Z^2 is to be obtained for the O and N-atoms.

We proceed from a general theorem by Thomas and Kuhn (see Bethe, *loc. cit.* p. 434):

$$\Sigma f_i + f_c = n, \qquad (17)$$

where Σf_i is the sum of oscillator-strengths for line radiation, f_c is the oscillator strength for the continuous radiation at the end of the series limit, the initial state being the normal state of the atom, 'n' is the number of equivalent electrons which can perform the transition in question. In the case of the O-atom, the photo-ionization can be represented symbolically as

$$1s^2\,2s^2.2p^4 \rightarrow 1s^2.2s^2.2p^3 \left.{\substack{\infty\,s\\ \infty\,d}}\right\}$$
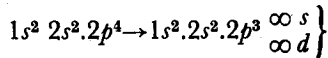
to find out f_c, we have to find out Σf_i, for

$$2p^4 \rightarrow 2p^3 \ (ms,\ md)$$

transitions and over all values of m from $m=3$ to $m=$ infinity.

This problem is difficult to solve, but it appears unlikely that Σf_i can be greater than unity. We can therefore put $f_c = 3$ for the O-atom, and 2 for the N-atom approximately. This view is consistent with that of Herzfeld and Wolf (1925) who showed, from a discussion of the dispersion curves of Ne, A and other rare gases, that for a proper interpretation of the course of dispersion of these gases, we have to suppose that the characteristic frequency in the Lorentz-expression for dispersion is not given by the resonance line of the element, but by a line which lies in the region of continuous absorption by the atom and that the number of equivalent electrons per atom for inert gases is nearly 5. In other words, when we have a large number of equivalent electrons in any shell, the tendency for ionization proportionately rises. We assume that when an atom contains r equivalent electrons in the outermost shells, $f_c = r - 1$ approx. provided r is large compared to unity. For helium, which contains two equivalent electrons, Vinti (1933) finds that $f_c = 1.52$.

Z is now obtained from the formula for oscillator-strength

$$f_c = \frac{g_i}{g_j}\frac{mc}{\pi e^2}\int_{\nu_0}^{\infty}\tau(\nu).d\nu. \qquad \cdots \qquad \cdots \qquad (18)$$

Substituting for $\tau(\nu)$ the value in (12),

$$f_c = \frac{g_i}{g_j}(.867)Z^2.$$

But there are further complications in the present cases. The normal oxygen atom has the electronic constitution $1s^2.2s^2.2p^4$ ($^3P_{210}\ {}^1D_2\ {}^1S_0$), and the oxygen-ion has the constitution $1s^2.2s^2.2p^3$ ($^4S_{3/2}D,\ {}^2P$). The transitions contemplated, which cause ionization, can take place from any state of the normal O-atom to that of any normal O-ion provided this is permissible, and in the calculation of f_c, all such separate transitions must be taken into account.

The continuous transitions fall into the following groups:—

$$
\begin{aligned}
O\,{}^3P \rightarrow O^+\ ({}^4S)_{sd} &\left.\begin{array}{l} \end{array}\right\}\ f_{c_1}\dots.I.P.\dots\dots 13.55\ \text{volts.}\\
\rightarrow O^+\ ({}^2D)_{sd}\ &\ f_{c_2}\dots\dots\dots\dots 15.86\dots\\
\rightarrow O^+\ ({}^2P)_{sd}\ &\ f_{c_3}\dots\dots\dots\dots 18.54\dots\\
O\,{}^1D_2\,{}^2 \rightarrow O^+\ ({}^2D)_{sd}\ &..f_{c_4}\dots\dots\dots\dots\\
O^+\ ({}^2P)_{sd}\ &\dots f_{c_5}\dots\dots\dots\dots\\
O\,{}^1S_0 \rightarrow O^+\ ({}^2P)_{sd}\ &..f_{c_6}\dots\dots\dots\dots
\end{aligned}
$$

The symbols (sd) denote that the hyperbolic orbit of the electron, which is released, may have $l=0$, or 2. This is necessary for calculation of the weight factors. We therefore expect (neglecting the small fine structure due to the presence of three ^3P-states) that there will be six distinct continuous absorption curves corresponding to the six processes mentioned above. In fact, we have

$$f_c = 3 = f_{c_1} + f_{c_2} + \dots\dots\dots f_{c_6}.$$

To calculate the relative values of the quantities f_{cr}, we can apply the arguments of Menzel and Goldberg (1936) about the parentages of the terms of the l^K-shell. According to these authors:—

$$\frac{f_{c_1}+f_{c_2}+f_{c_3}}{9(=g\ \text{of}\ {}^3P)} = \frac{f_{c_4}+f_{c_5}}{5(=g\ \text{of}\ {}^1D_2)} = \frac{f_{c_6}}{1(=g\ \text{of}\ {}^1S_0)} = \frac{3}{15}$$

and

$$f_{c_1} : f_{c_2} : f_{c_3} = 4 : 10 : 6.$$

Hence $f_{c_1} = \frac{9}{25}$, $f_{c_2} = \frac{9}{10}$, $f_c = \frac{27}{50}$.

We have therefore from (16) for the process $O\,{}^3P \rightarrow O^+\,{}^4S$

$$Z^2 = \frac{g_j}{g_i}\frac{f_{c_1}}{.867} = \frac{48}{9}\frac{9}{25 \times .867} \qquad \cdots \qquad \cdots \qquad (19)$$

because

$$g_i = \text{weight factor of}\ O\,{}^3P_{210} = 9$$
$$g_j = \text{weight factor of}\ (O^{+4}S)_{sd} = 48$$

we get $Z = 1.49$.

The value of τ_0 for the process $O\,{}^3P \rightarrow O^+\,{}^4S$ now becomes

$$\tau_0 = \frac{2^8}{3}.\frac{\pi e^2}{mc}.\frac{1}{\nu_0}\epsilon^{-4}.Z^2 = 2.81 \times 10^{-17}.cm^2$$

The value given by Chapman is $2.5 \times 10^{-16}\ cm^2$, i.e. nearly nine times larger.

We can, in a similar way, find out the effective values of Z for the processes

$$O\,{}^3P \rightarrow O^+\ ({}^2D)_{sd}.\ \ O\,{}^3P \rightarrow O^+\ ({}^2P)_{sd}.$$

The g_j-value of $(O^+ {}^2D)_{sd}$ state is 120, and that for $(O^{+\cdot 2}P)_{sd}$ is 72.

The corresponding Z-values are 3·04 and 2·36 respectively.

There will certainly be some difficulty in following why the effective nuclear charge in O should be different for the three processes mentioned above. But actually there is no difficulty as can be seen from the argument that $\tau(\nu)$ is given by the value of the transition-probability, from a certain initial (lower) state to a final state. Hence it will involve the effective nuclear charge for the initial as well as the final state. In fact, Z^2 should be replaced by $Z_i \, Z_f$. In these cases, though Z_i is identical, Z_f is different, as the electrons of the ion have different configurations, when giving rise to the different terms. Further, as it is a question of transition-probability, Z^2 may be widely different.

We have not calculated Z-values for transition from $O\ {}^1DS$ to $O^+\ {}^2DP$-states, as probably there are not sufficient $O\ {}^1DS$-atoms in the ionosphere capable of producing any sensible ionization, for $O{}^1DS$-atoms produced by any photo-electric process almost instantaneously revert to $O\ {}^3P$-state.

The Nitrogen-atom

These considerations may now be extended also to the nitrogen-atom. We have now

$$f_c = 2.$$

The f_c-value is distributed as follows:—

$$N\ 2p^3.{}^4S \rightarrow N^+\ 2p^2({}^3P)_{sd}\ldots f_{c_1}\ldots I.P.\ldots 14{:}46 \text{ volts}$$
$${}^2D \rightarrow N^+\ (2p^2\ {}^3P)_{sd}\ldots f_{c_2}\ldots\ldots\ldots\ldots$$
$$N^+\ (2p^2\ .\ {}^1D_2)_{sd}\ldots f_{c_3}\ldots\ldots\ldots\ldots$$
$${}^2P \rightarrow N^+\ (2p^2\ .\ {}^3P)_{sd}\ldots f_{c_4}\ldots\ldots\ldots\ldots$$
$$N^+\ (2p^2\ .\ {}^1D_2)_{sd}\ldots f_{c_5}\ldots\ldots\ldots\ldots$$
$$N^+\ (2p^2\ {}^1S_0)_{sd}\ldots f_{c_6}\ldots\ldots\ldots\ldots$$

We have

$$\frac{f_{c_1}}{4} = \frac{f_{c_2}+f_{c_3}}{10} = \frac{f_{c_4}+f_{c_5}+f_{c_6}}{6} = \frac{f_c}{20}$$

and $\qquad f_{c_2} : f_{c_3} = 9 : 5 : f_{c_4} : f_{c_5} : f_{c_6} = 9 : 5 : 1.$

Hence we have

$$f_{c_1} = ·4, \quad Z = 3·53.$$

We need not calculate the Z-values for the other states of nitrogen, as practically all N-atoms will be in the ${}^4S$-state.

5. Rate of Production of Electrons by Photo-ionization

We shall now use the expression for $\tau(\nu)$ in formula (10) for the calculation of q according to the expression (6). We observe that if p be the partial pressure due to the absorbing particles at the region considered

$$I(\nu) = I(\nu)_0 \operatorname{Exp}\left\{ -\frac{p\tau(\nu)}{Mg} \sec \chi \right\}, \quad (20)$$

when radiation is incident at an angle χ to the vertical, and $I(\nu)_0$ is the intensity of light just outside the earth's atmosphere. The relation (20) is proved as follows (first given by Pannekoek, 1926). On passing through a layer having the thickness dz (the layers are supposed to be parallel), the diminution in intensity is given by

$$dI(\nu) = -I(\nu)\ \mathcal{N}\tau(\nu).dz.\sec \chi. \quad (21)$$

We have further the hydrostatic equation

$$dp = \mathcal{N}gM\ dz, \text{ i.e., } \mathcal{N}dz = dp/gM. \quad (22)$$

Here M is the mass of each particle, and p is reckoned from the top of the atmosphere.

From (21) and (22) we have

$$\frac{dI(\nu)}{I(\nu)} = -\frac{\tau(\nu)}{gM} dp \ \sec \chi.$$

and on integrating this, we arrive at expression (20). It is easy to verify that for an isothermal layer, (20) reduces to (2), but (20) is more general. Substituting this in (6′).

$$q = \mathcal{N}. \int_{\nu_0}^{\infty} I(\nu)_0 \cdot \psi(\nu) \cdot \operatorname{Exp}\left\{ -\frac{p\tau(\nu)}{Mg} \cdot \sec \chi \right\} d\nu. \quad (23)$$

It is found rather difficult to integrate (23) rigorously on account of complexity of its form.

We can replace $\tau(\nu)$ within the integral (23) in the exponential by its mean value $\frac{5}{9}\tau_0$, where τ_0 is the value of $\tau(\nu)$ at the threshold frequency. We have then, taking the exponential term outside,

$$q = \frac{p}{kT} \operatorname{Exp}\left\{ -\frac{5}{9} \cdot \frac{p\tau_0}{Mg} \cdot \sec \chi \right\} \cdot \int_{\nu_0}^{\infty} I(\nu)_0\ \psi(\nu) \cdot d\nu.$$

Now as we have

$$I(\nu)_0 = \frac{8\pi\beta \cdot h\nu^3}{c^2} e^{-\frac{h\nu}{kT_s}}, \quad \psi(\nu) = \tau(\nu)/h\nu,$$

where $4\pi\beta$ is the solid angle subtended by the sun at the earth, $\beta = \dfrac{1}{230,000}$, T_s=temperature of the sun. The integral reduces to

$$\frac{8\pi\beta}{c^2} \int_{\nu_0}^{\infty} \nu^2 \cdot e^{-\frac{h\nu}{kT_s}}\ \tau(\nu) \cdot d\nu.$$

For $\tau(\nu)$, we can write the approximate value (12). Hence the integral

$$= \frac{8\pi\beta}{c^2} \cdot \frac{\tau_0}{3}\nu_0^3 \int_1^{\infty} \left(\frac{4}{x} - \frac{1}{x^2} \right) e^{-\mu_1 x} \cdot dx,$$

where $\mu_1 = \dfrac{h\nu_0}{kT_s}$. If we put $T_s = 6240°\mathrm{K}$ and ν_0=ionization threshold frequency for the oxygen atom, it can be easily

seen that $\mu_1 = 25 \cdot 78$ and for such large values of the index, it is easy to show that

$$\int_1^\infty \left(\frac{4}{x} - \frac{1}{x^2}\right) e^{-\mu_1 x} \cdot dx = \frac{3e^{-\mu_1}}{\mu_1}.$$

Hence we have

$$q = \frac{8\pi\beta}{c^2h} \cdot \tau_0 \cdot v_0^2 \cdot T_s \cdot e^{-\frac{h v_0}{kT_s}} \cdot \frac{p}{T} \cdot \mathrm{Exp}\left\{-\frac{5}{9}\frac{p\tau_0}{Mg}\sec\chi\right\}$$

$$= A \cdot \frac{p}{T} \cdot \mathrm{Exp}\left\{-\frac{p \cdot \sec\chi}{p_0}\right\}, \qquad \qquad .. \quad (24)$$

where

$$\left.\begin{array}{l} A = \dfrac{8\pi\beta}{c^2h} \cdot \tau_0 \cdot v_0^2 \, T_s \cdot e^{-\frac{h v_0}{kT_s}} \\[2mm] p_0 = \dfrac{9}{5} \cdot \dfrac{Mg}{\tau_0} \end{array}\right\} \qquad (25)$$

We can now try to find out the maximum value of q. Differentiating (24), we have

$$\frac{1}{q} \cdot \frac{dq}{dz} = \frac{dp}{dz}\left(\frac{1}{p} - \frac{\sec\chi}{p_0}\right) - \frac{1}{T}\frac{dT}{dz} \quad \cdots\cdots$$

Let us first assume that $\dfrac{dT}{dz} = 0$, though we cannot be sure of the truth of this assumption. We have then

$$\frac{dq}{dz} = 0, \text{ when } p = \frac{9}{5}\frac{Mg}{\tau_0}\Big/\sec\chi = p_0 \cos\chi. \quad .. \quad (26)$$

p_0 is the partial pressure at the place where we have maximum production of ions under vertical incidence. For incidence at an angle χ to the vertical, the maximum concentration is reached at the pressure $p = p_0 \cos\chi$.

Inserting this value of p in (24), we have for q_m, the maximum production of ions at angle χ, the expression

$$q_m = q_0 \cos\chi, \qquad \qquad .. \qquad .. \quad (27)$$

where q_0=maximum production of the ions under vertical incidence, and it is given by

$$q_0 = \frac{A}{T} \cdot \frac{9}{5} \cdot \frac{Mg}{\epsilon\tau_0} = \frac{8\pi\beta}{c^2h} \cdot \frac{9}{5} \cdot \frac{Mg}{\epsilon} \frac{T_s}{T} v_0^2 e^{-\frac{h v_0}{kT_s}}. \quad (28)$$

We observe that q_0 is independent of τ_0. This is because the smaller is the value of τ_0, the higher is the partial pressure where maximum ionisation is reached.

The value of q, at a point where the pressure is p, is given by

$$q = q_0 y \, \mathrm{Exp}\left\{1 - y \sec\chi\right\}, \qquad .. \quad (29)$$

where $y = p/p_0$.

It can be easily shown that (29) yields us Chapman's expression for electron-production, if p is supposed to be given by the isothermal law

$$p = P\,\mathrm{Exp}\left\{-\frac{h}{H}\right\},$$

where $H = \dfrac{kT}{Mg}$=height of the homogeneous atmosphere, P=ground pressure.

For we can put, following Chapman,

$$p_0 = P.\,\mathrm{Exp}\left\{-h_0/H\right\}, \quad z = (h - h_0)/H$$

so that $y = p/p_0 = e^{-z}$.

Then (29) becomes

$$q = q_0\,\mathrm{Exp}\left\{1 - z - e^{-z}.\sec\chi\right\}. \quad .. \qquad .. \qquad .. \quad 30$$

This is the expression given by Chapman for variation of production of ions with height.

We have thus proved that even when photo-ionization is produced by continuous light, variation of ion-production with height continues to be given by Chapman's formula when T is supposed to be constant. Chapman proved the correctness of (30) for monochromatic light only. Further, and this is a definite advance on Chapman's result, we have now a perfectly definite expression for q_0 which is left undefined in Chapman's method of deduction.

We can now try to give some numerical calculations for q_0 and p_0 for the oxygen atom. As shown before, we have for the process

$$O\ ^3P \to O^+\ ^4S, \quad \mathcal{Z} = 1 \cdot 49, \quad v_0 = 109837\ c.$$
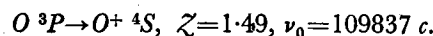
Substituting these values in (25), we obtain

$$A = 3 \cdot 71 \times 10^8, \quad p_0 = \frac{9}{5}\frac{Mg}{\tau_0} = 1 \cdot 65 \times 10^{-3}\ \text{dynes}.$$

$$q_0 = \frac{1 \cdot 35 \times 10^5}{T}. \qquad \qquad .. \quad (31)$$

We have assumed that $T_s = 6240°\mathrm{K}$.

Appleton (1938) has calculated from the maximum electron-concentration curve that at Slough for an equinoctial day, when $\chi = 52°$, $q_m = 78$, and for a midsummer day, i.e. for $\chi = 29°$, $q_m = 88$. These values may be compared with those available from (31).

We have

$$q = q_0 \cos\chi = \frac{1 \cdot 35 \times 10^5 \cos\chi}{T}$$

$$= 8 \cdot 3 \times 10^4/T \quad .. \qquad .. \quad \text{for equinoctial noon}$$

$$= 11 \cdot 8 \times 10^4/T \quad .. \qquad .. \quad \text{for midsummer noon}.$$

If we take T, the temperature for the F_2-layer at noon $1065°\mathrm{K}$, for an equinoctial day, and $1350°\mathrm{K}$ for a midsummer day, the above results are explained. These values are not inconsistent with the values now assigned to the temperature of the Upper Atmosphere (Appleton, 1935).

We have given these results with a certain amount of reserve; it must not be supposed that we commit ourselves to the opinion that the F_2-layer is due solely to the ionization of O-atoms. We merely wish to point out that our calculations give us the right order of result.

Formula (7) gives us the recombination coefficient α. We can now neglect $I(\nu)$ in comparison to $8\pi h\nu^3/c^2$, and put for $\tau(\nu)$ the expression (12). We obtain after some work, for recombination between $O^+ \, ^4S$ and e,

$$\alpha = \frac{h}{(2\pi m)^{3/2}k^{\frac{1}{2}}} \cdot \frac{8\pi}{c^2} \cdot \tau_0 \, \nu_0^2 \cdot \frac{1}{T^{\frac{1}{2}}} \qquad \cdots \qquad \cdots \quad (32)$$

when we introduce the values of τ, ν_0, for the process

$$O^+ \, ^4S + e = O \, ^3P, \quad \alpha = \frac{1\cdot 25 \times 10^{-10}}{T^{\frac{1}{2}}} = 6\cdot 3 \times 10^{-12}.$$

provided $T = 400°K$. This value may be compared with the figures given by Appleton (1938), whose average values are nearly $10-20$ times larger.

6. Equilibrium Value of Electron Concentration

The equilibrium value of electron concentration can now be easily calculated by putting for $\tau(\nu)$ the expression (12), in formula (8). We have now

$$K = K_0 \frac{\beta \int_1^\infty \left(\frac{4}{x} - \frac{1}{x^2}\right) e^{-\mu_1 x} \text{Exp}\left\{-\frac{p\tau_0}{3Mg}\left(\frac{4}{x^3} - \frac{1}{x^4}\right)\sec\chi\right\}}{\int_1^\infty \left(\frac{4}{x} - \frac{1}{x^2}\right) e^{-\mu_2 x}\, dx},$$

where $\mu_1 = \frac{h\nu_0}{kT_s}$, $\mu_2 = \frac{h\nu_0}{kT}$, $x = \nu/\nu_0$.

Now both μ_1 and μ_2 are large quantities. In fact, as we have seen, if we take $T_s = 6240°K$, $\mu_1 = 26\cdot 9$, and for $T = 1000°K$, $\mu_2 = 161\cdot 4$.

The quantity

$$\text{Exp}\left\{-\frac{p\,\tau_0}{3Mg}\sec\chi\left(\frac{4}{x^3} - \frac{1}{x^4}\right)\right\}$$

within the upper integral can be replaced by its mean value

$$\text{Exp}\left(-\frac{5}{9}\frac{p\,\tau}{Mg} \cdot \sec\chi\right),$$

and taken outside as already described.

We can then easily show that

$$K = K_0\beta \cdot \frac{\mu_2}{\mu_1} e^{-(\mu_1 - \mu_2)} \, \text{Exp}\left\{-\frac{5}{9}\frac{p\tau_0}{Mg}\sec\chi\right\}$$

$$= \beta T_s T^{\frac{1}{2}} e^{-\frac{h\nu_0}{kT_s}} \left(\frac{2\pi mk}{h^2}\right)^{3/2} \text{Exp}\left\{-\frac{5}{9}\frac{p\,\tau_0}{Mg}\sec\chi\right\}, \quad (33)$$

which has the same form as the result deduced by Pannekoek. We have the factor 5/9 instead of Pannekoek's $\frac{3}{4}$. From this expression, we can easily calculate the

maximum equilibrium value of electron concentration. We have according to (1)

$$K = \frac{n_+ n_e}{N} = \frac{q}{\alpha N},$$

and the last relation can be easily verified by reference to formulae (24) and (32) for q and α. If $n_+ = n_e$, as will happen in the case of unicomponent systems, we have $n_e = (q/\alpha)^{\frac{1}{2}}$. From this, it is clear, as α involves T only, and is independent of p, that in an isothermal atmosphere, the maximum value of n_e will occur at the same place as that for q. It is now easy to show that

$$\left.\begin{aligned} n_0 &= (q/\alpha)^{\frac{1}{2}} = \frac{2\cdot 55 \times 10^7}{T^{1/4}} \\ n_m &= n_0(\cos\chi)^{\frac{1}{2}} \\ n_e &= n_m y^{\frac{1}{2}} \text{Exp}\,\tfrac{1}{2}\{1 - y\sec\chi\} \\ \text{where } y &= p/p_0. \end{aligned}\right\} \quad \cdots \quad (34)$$

The variation of electron-density in the E and F_1-layer is found to be given by the law $n = n_0(\cos\chi)^{\frac{1}{2}}$ during hours of daylight. This is often cited (see Appleton, 1938) as proof of the correctness of Chapman's theory of simple region formation. But formulae (30) and (35) show that this relation holds good even when electrons are produced by continuous light.

7. Ionization by Monochromatic Light

In this section, we shall give a treatment of the ionization produced by monochromatic light, because it has been pointed out by several investigators that many upper air phenomena are probably due to ionization by monochromatic light from the sun. Maris and Hulburt (1929) talks of flares of ultraviolet radiation to account for magnetic storms, and abnormal display of aurora. One of us (Saha, 1935) has pointed out that the strong N_2^+-ionization observed by Slipher (1933), in the morning and evening flash of sunlight in the Upper Atmosphere, is due to photo-ionization of N_2 due to emission lines of $He\, 1s^2\, ^1S_0 - 1smp\, ^1P$. Dellinger (1937) and others have tried to connect the sudden bursts of ionization which give rise to radio fade-outs in the sunlit part of the globe to the occurrence of disturbed regions on the sun showing strong H_α and H_β-lines in emission (vide further a note by R. N. Rai and K. B. Mathur, 1937).

The number of electrons produced by a monochromatic beam of intensity $I(\nu)$ is given by

$$q = N.I(\nu).\psi(\nu).\Delta\nu.$$

Here $\Delta\nu$ is the equivalent breadth, N is the number of atoms or molecules in the region. Then we have

$$\text{since } I(\nu) = I(\nu)_0 \text{Exp}\left\{-\frac{p\,\tau(\nu)}{Mg} \cdot \sec\chi\right\},$$

$$\text{and } N = p/kT$$

$$q = A' \frac{p}{T} \text{Exp} \left\{ -\frac{p\,\tau(\nu)}{Mg} \sec \chi \right\},$$

where $A' = \dfrac{I(\nu)_0 \psi(\nu).\Delta\nu.}{k}$.

This expression is exactly similar to the expression (30) for ion-production by continuous light, but is considerably simpler. Here

$$A' = \frac{I(\nu)_0 \Delta\nu\,\tau(\nu)}{k\,.\,h\nu}$$

is proportional to number of quanta absorbed per atom, and $\tau(\nu)$ has a perfectly definite value. The value of $I(\nu)_0$ depends upon the intensity of the light which may be available from astrophysical measurements, and the dimensions of the disturbed area on the sun. In case the disturbance extends over the whole surface, $I(\nu)_0$ is proportional to β but otherwise it will be much less, and will be equal to the solid angle subtended by the disturbed region in the earth's atomsphere.

We can find out q, q_m, q_0 as defined in sec. 5 exactly in the same way as there, and we have for an isothermal atmosphere

$$\left.\begin{array}{l} q_m = q_0 \cos \chi \\[1mm] q_0 = \dfrac{A'}{T}\dfrac{gM}{\epsilon\tau(\nu)} = \dfrac{I(\nu)_0\,\Delta\nu}{h\nu}\dfrac{gM}{\epsilon kT} \\[2mm] q = q_0 y \, \text{Exp}\,\{1 - y \sec \chi\} \\[2mm] y = p/p_0, \text{ where } p_0 = \dfrac{gM}{\epsilon\,\tau(\nu)} \end{array}\right\} \quad \cdots \quad \cdots \quad (35)$$

Attention may be drawn to the expression for q_0, the maximum electron-production per c.c. under vertical incidence—it is equal to the total number of ions produced by the total absorption of the beam divided by ϵH, where H is the height of the homogeneous atmosphere. This relation is identical with that obtained by Lenard and Chapman.

ABSTRACT

It has been shwon that the two theories of upper air ionization, viz. that of Pannekoek and Chapman, are not essentilly different from each other. When in the Chapman theory we give to the qaunfity β, which is introduced as a proportionality factor for deducing the number of electrons from the radiation absorbed, the value $\dfrac{1}{h\nu}$, and extend it to continuous radiation, we come to Pannekoek's results.

For absorption coefficient $\tau(\nu)$, a wave mechanical formula is used in the place of Kramers-expression. Rates of production of electrons from the O-atoms, the recombination co-efficient of ions and electrons, and equilibrium values of electron concentration are deduced for unicomponent systems. From these expressions, Chapman's formulæ for variation of electron-production with height is deduced as a special case but the scope of the formula is found to be greater as it is found to hold not only for monochromatic light, as in Chapman's, but also for continuous light. Actual values of electron production at noon for the F-layer from the O-atom are given, and compared with figures given by Appleton.

It is further shown that the method is capable of giving also formula for electron production by monochromatic light, and yields results in terms of quantities which are physically definable.

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