8. ON RADIATION-PRESSURE AND THE QUANTUM THEORY

A PRELIMINARY NOTE

(*Astrophys. Journ.*, **50**, 220, 1919.)

After the prediction by Maxwell of the existence of the pressure of radiant energy on the basis of his theory of stresses and strains in ether, other ways of arriving at the same result have been found by Bartoli (thermodynamical), Poynting (flow of momentum along a ray of light), and Larmor (electromagnetic wave-theory of light). A review of these methods shows that they are all statistical, i.e., the result holds only when the surface encountered by radiation is large compared with the wave of light and is thickly set with matter.

Schwarzschild and more recently Nicholson[1] and Klotz[2] have worked out, on the basis of the continuous theory, the value of the radiation-pressure, when the size of the obstructing mass is gradually decreased, ultimately being reduced to the scale of the wave-length of light. In this case the effect of repulsing light-pressure gradually preponderates over any gravitative force to which the particle may be subject, but at the same time it appears that there is a limit to this process of reduction. If the particle be too small, it is no longer capable of acting as a barrier to the advancing light-waves, and consequently experiences no radiation-pressure. It appears from these investigations that for particles of the molecular size (radius$=10^{-8}$ cm) the effect of light-pressure is totally evanescent.

But this conclusion from the old continuous theory is rather contradictory to the requirements of astrophysics, for in order to explain tails of comets and other astrophysical phenomena (such as solar prominences, corona) which take place on the surface of luminous heavenly bodies we have to assume the existence of certain repulsive forces[3] (levity) acting on the ultimate gaseous molecules and thus reducing the gravitational attraction on them. But a still stronger ground for rejecting the conclusion is furnished by the experimental demonstration by Lebedew[4] of the existence of radiation-pressure on molecules of absorbing gases like CO_2, methane, propane, etc. It may thus be taken for granted, in spite of the failure of the continuous theory, that the molecules do really suffer a radiation-pressure, which in the aggregate conforms to Maxwell's law.

Professor R. W. Wood[5] is inclined to the opinion that the gas molecule may be capable of stopping the radiation by resonance, and may thus experience a radiation-pressure, but precisely what is meant by stoppage of radiation by resonance is not clear. An explanation of the existence of radiation-pressure on molecules is furnished when we apply the quantum theory in the place of the old continuous theory of light. Instead of assuming that "light" is spread continuously over all points of space, let us suppose that they are localized in pulses of energy $h\nu$ (ν=frequency of light, h=Planck's universal radiation constant). Let this pulse encounter a molecule m and be absorbed by it. Then in doing so the molecule will be thrust forward with an impulsive momentum of $\frac{h\nu}{c}$ (c= velocity of light); for we may suppose the pulse to have the mass $\frac{h\nu}{c^2}$ and the momentum $\frac{h\nu}{c}$; the absorption of the pulse by the molecule may be taken as a case of inelastic impact, the whole momentum being communicated to the molecule. The velocity with which the molecule will move forward$=\frac{h\nu}{cm}$.

Let us consider the effect of the absorption of a pulse of the hydrogen light corresponding to the line $H\alpha$ by the hydrogen atom. The velocity imparted at each kick of light

$$v=\frac{h\nu}{cm}=60 \text{ cm per second,}$$

$$\left(\text{taking } h=6.54\times 10^{-21}\right.$$

$$\left.\frac{c}{\nu}=\lambda=6.563\times 10^{-5}\text{cm.};\ m=\frac{1}{6.062\times 10^{23}}\text{ gms}\right).$$

This velocity is rather a small quantity (compared to the orbital velocity of the molecules), but it should be remembered that it is really an impulsive velocity and is of the nature of an acceleration. The total velocity acquired by a hydrogen atom per second will depend upon the number of kicks of light it experiences per second, and provided this is sufficiently great the velocity acquired may rise to enormous values. But a priori we cannot say what this number will amount to without a preliminary examination of the physical conditions.

This conclusion explains Lebedew's results, which cannot be explained by the continuous theory, and at the same

[1] *Monthly Notices*, **74**, 425, 1914.
[2] *Journal of the R.A.S. of Canada*, **12**, 357, 1918.
[3] See Agnes M. Clerke, Problems of Astrophysics, p. 51.
[4] *Annalen Physik*, **92**, 411, 1910.
[5] Physical Optics, p 51.

time offers a general explanation of the radiation-pressure. The pressure $=\frac{1}{c}\Sigma\Sigma h\nu$, where the summation extends over all the pulses absorbed in unit time, within unit area. It thus equals AI, where I=intensity of light, A=fraction absorbed. The aggregate effect remains unchanged, but it is now supposed to be concentrated on a few active molecules, the inactive molecules remaining unaffected.

The explanation offered closely resembles Einstein's explanation of the velocity of emission of the photo-electrons. According to Einstein when a pulse of light ($h\nu$) falls upon an atom it is instantly absorbed and goes to increase the energy of the system. Consequently certain of the electrons of an atomic system acquire a velocity which is greater than the critical velocity required for retaining these electrons in their orbit. Let A be the energy required for detaching an electron from the parent atom. Then the velocity of escape is given by the law

$$\Sigma\tfrac{1}{2}mv^2=h\nu-\Sigma A.$$

The maximum velocity occurs when only one electron is emitted. Then

$$\tfrac{1}{2}mv^2=h\nu-A.$$

Actual experiments by Millikan[6] and others have established the truth of the law quantitatively. Besides, the phenomenon is instantaneous whatever be the intensity of the light. This feature is not capable of explanation by the continuous theory of absorption. N. R. Campbell[7] has found that in certain cases the continuous theory requires that the atom must be illuminated for at least 15 minutes before it can acquire the energy sufficient for the emission of the electron, while actually the emission takes place in less than $\frac{1}{1000}$ of a second after illumination.

Let us now see how the number of kicks of light experienced by the hydrogen atom or molecule varies with the existing circumstances. The number will clearly depend upon the following factors: (1) the density of pulses of light in the region traversed by the molecule; (2) the time of retention by the molecule or the atom of the capacity for the absorption of light. We shall first take the second point. Hydrogen under ordinary circumstances does not absorb its characteristic radiation (represented by the Balmer lines), as has been demonstrated by the repeated failures of the experiments for obtaining the reversal of the hydrogen lines. But the experiments of Ladenburg and Loria[8] have thrown a new light on the cause of these failures; they find that hydrogen is capable of absorbing its characteristic radiation only when it is in an active state, i.e., when it is in a state of luminescence. This conclusion is also borne out by the theoretical investigations of Bohr[9], for according to his theory a hydrogen line is emitted when the attendant electron leaps from orbit 3 to orbit 2, while in the natural state the electron is at orbit 1. We may symbolically express the idea as in Fig. 1.

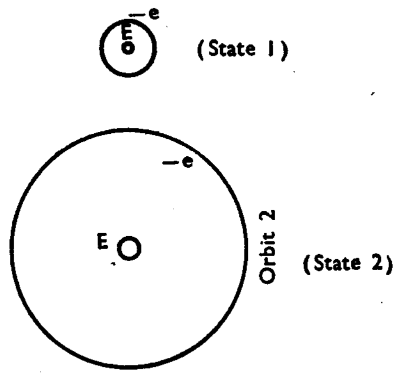

Fig. 1

State 1, natural state when inactive for the Balmer lines.

State 2, active state (when emitting the Balmer lines).

In order that an H atom may absorb a Balmer line, it must be, to start with, at state 2.

We may thus take it for granted that the H atoms which absorb the Balmer lines are not the ordinary H atoms, but an active modification thereof, the electron being at orbit 2 instead of at orbit 1. When light corresponding to any line of the Balmer spectrum traverses a mass of hydrogen, it is only the active particles which will absorb this light and be subjected to the impulsive kicks of this light.

Taking it for granted that an active molecule suffers a discontinuous kick of light given by the formula in the process of absorption, let us see how it will behave when placed in a field of radiation. To visualize matters, we shall take an active H atom moving near the photosphere of the sun. The H atom, if active to start with, will pick out from the continuous spectrum the pulse corresponding to $H\alpha$ or $H\beta$ and with an instantaneous velocity of 60 to 31 cm per second. It is true that, as the particle emits light, it suffers an equal recoil opposite to the direction of emission. It should be borne in mind that the emission does not take place in any specified direction, but in any direction according to the law of chance, while the pulses which are absorbed come from a specified direction, viz., the center of the sun. Hence if the particle continues active for a sufficient length of time, the H atom may ultimately acquire a velocity exceeding the critical velocity of 6.12×10^7 cm per sec. (the velocity required for the escape of the particle from the gravitational influence of the sun).

The precise velocity which a particle acquires depends upon a large number of unknown factors: (1) the intensity of the field of radiation—the influence of this factor is to a certain extent known—the density of pulses varies as the intensity of light, and therefore follows the law of inverse square; (2) the persistence of the activity of the H atom, or rather, if the activity be lost, the quickness with which it is regenerated; (3) the actual proportion of active particles in any region.

Nothing is known about the second and the third factors; consequently it is not possible to work out a quantitative

[6] *Physical Review*, **7**, 18, 1916.
[7] Modern Electrical Theory, p. 249.
[8] *Verhandlungen der deutschen Physikalischen Gesellschaft*, **10**, 858, 1908.
[9] *Philosophical Magazine*, **26**, 1, 476, 857, 1913.

theory of the effect of radiation-pressure on the expulsion of the molecules. But the general considerations show *that radiation-pressure may exert an effect on the atoms and molecules which are out of all proportion to their actual sizes. It also shows that the radiation-pressure exerts a sort of sifting action on the molecules, driving the active ones radially outward along the direction of the beam. The cumulative effect of the pulses may be sufficiently great to endow the atoms with a large velocity—the velocity with which the tops of solar prominences are observed to shoot up.*

The velocity of the red prominences are sometimes found to be as high as 6×10^7 cm per second.

The solar prominences have sometimes been explained on the assumption that they are due to the convection of hot masses of vapor from the solar photosphere, which, after reaching the atmosphere, are supposed to expand adiabatically and develop the large velocities with which the prominences are observed to shoot up. But both Pringsheim and Nicholson[10] have pointed out several insuperable difficulties in the way of the acceptance of this hypothesis, including the deduction that the maximum velocity obtainable from adiabatic expansion is less than $\frac{1}{25}$ of the velocity with which the prominences are observed to shoot forward (6×10^7 cm). Nicholson has suggested that some unknown forces of electrical origin may be the cause of these large velocities, but even granting that the electrical fields exist in the sun it is difficult to see how this can act upon the luminous hydrogen particles, which are most probably uncharged. According to the hypothesis put forward in this paper, the effect of radiation-pressure on the separate particles is altogether disproportionate to the dimensions of the particles and may cause them to be endowed with a "levity"[11] long sought for in the explanation of the prominences, the corona, and other solar phenomena, including the extension of the solar atmosphere.[12] The hypothesis presents the problem of the radiative equilibrium of the solar atmosphere in a new light.

These ideas may be applied to the explanation of the tails of comets. The tails of comets are undoubtedly caused by some sort of repulsive action exerted by solar light, but since, on the older theory, the effect was found evanescent on particles of the molecular size, the tail was supposed to consist of some sort of cosmic dust. But the spectroscopic examination of the light from the tails shows that they consist, at least partly, of luminous gases (CO, CO_2)[13]. Now the explanation is quite easy, if the considerations advanced in this paper hold. As the comet approaches the sun, more and more pulses of light from the sun traverse the nucleus and the coma. Light-pulses of suitable frequency are picked up by the gaseous particles, which thus gradually gain in velocity in a direction away from the sun. The cumulative effect of the absorbed pulses may endow the particle with a velocity sufficient for its escape from the main mass of the cometary matter and form into the tail.

It is hoped to develop these ideas further in a future communication.

UNIVERSITY COLLEGE OF SCIENCE, CALCUTTA.
MARCH 4, 1919.

[10] *Monthly Notices*, **74**, 425, 1914.

[11] Ch. Fabry, lecture delivered before the Astronomical Society of France, 1918 (*L'Astronomie*, **32**, 14, 1918).

[12] Attention may be called to a comprehensive paper by D. Brunt (*Monthly Notices*, **73**, 568, 1913), who has shown that neither of the three theories of the equilibrium of the solar atmosphere (isothermal, adiabatic, or radiative) can account for an atmosphere extending to the observed height of the solar atmosphere. The results of the spectroheliographic observations are distinctly unfavorable to Julius' theory of anomalous dispersion (see *Astrophysical Journal*, Papers by Hale, St. John, and others).

[13] Bohr, *loc cit.*

9. ON THE FUNDAMENTAL LAW OF ELECTRICAL ACTION[1]

(*Phil. Mag., Sr. VI*, **37**, 347, 1919)

1.

In the present paper an attempt has been made to determine the law of attraction between two moving electrons, with the aid of the New Electrodynamics as modified by the Principle of Relativity. The problem is a rather old one, and seems to have first occurred in 1835 to Gauss[2], from whom the title of the paper has been borrowed. Before explaining my methods, I shall give a short history of the problem.

About the year 1826 Ampère published his celebrated laws of electrodynamic action, which enable us to calculate, with strict mathematical exactness, the action between two closed electric currents. If we assume that a current of electricity consists of streams of positive and negative charges moving in opposite directions, this action between two closed currents is seen to be composed of the elementary actions between the moving charges, taken two and two. The moving charges, therefore, cannot attract or repel in the same manner as two stationary charges (viz. force $=ee'/r^2$), for in that case the total action would be zero.

[1] Communicated by Prof. D. N. Mallik.

[2] Much of the Introduction is taken from Maxwells' 'Electricity and Magnetism', Chaps. II and XXIII, see especially pp. 483 *et seq.*