Lecture 1: cross section and reaction yield
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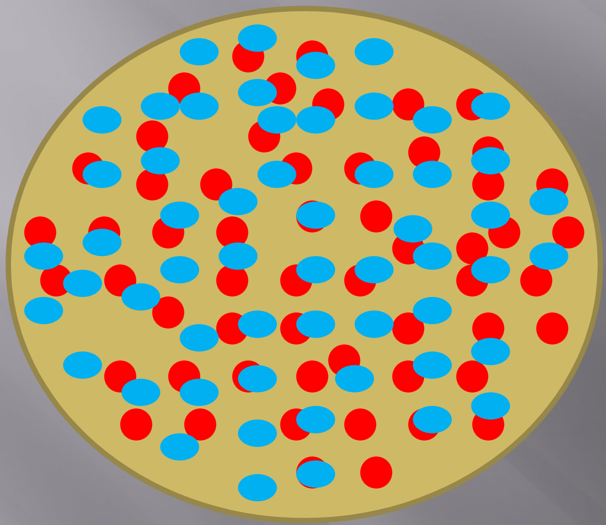
Nuclear reactions in stars

$X + a$



if v is the relative velocity

$$Y = N_a N_X \sigma v$$

but v has a distribution P(v)

$$Y = \int_0^\infty N_a N_X \sigma P(v) v dv = N_a N_X \langle \sigma v \rangle$$

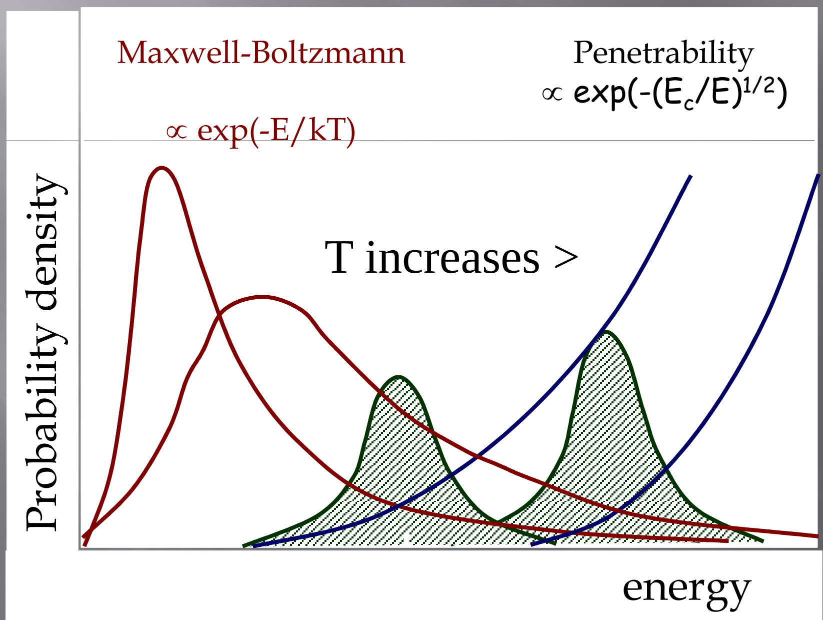
P(v) -> Maxwell Boltzmann distribution

$$\langle \sigma v \rangle = \left(\frac{8}{\pi}\right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^{\infty} \sigma(E)\, E \exp\left[-\frac{E}{kT}\right] dE$$

σ > if non resonant, dominated by the penetrability of the coulomb barrier

$E_0 = f(Z_1, Z_2, T)$

Note:resonances may shift the relevant energy in stars



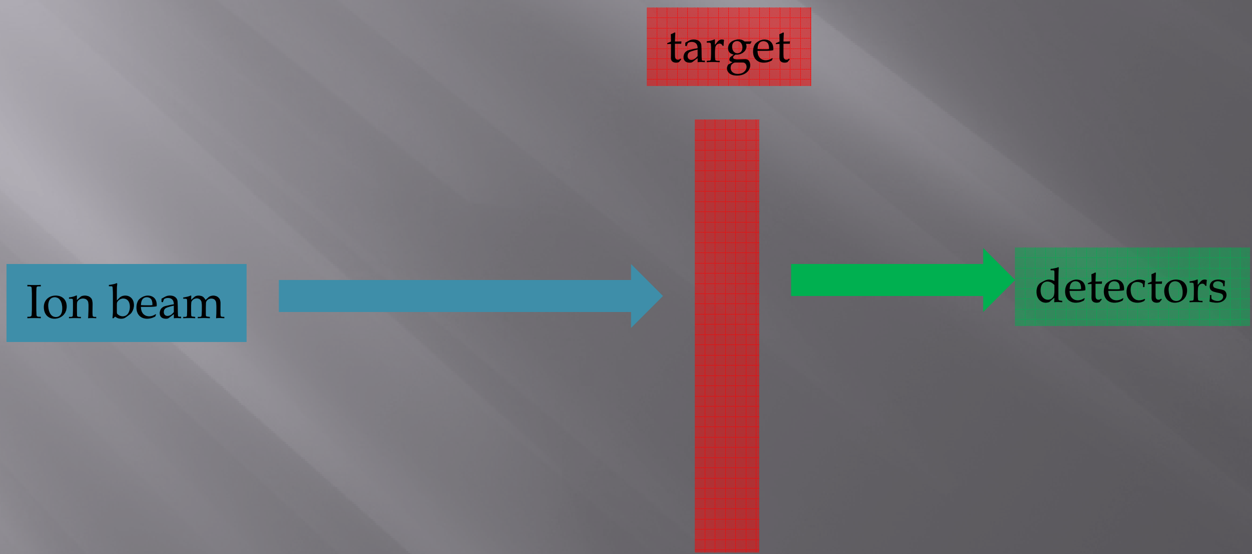
This is the reason for separate, subsequent burnings

Sun : $T_6 = 15$

reaction	E_0(keV)	Integral
p+p	5.9	$7\ 10^{-6}$
$\alpha + {}^{12}C$	56	$5.9\ 10^{-56}$
${}^{16}O + {}^{16}O$	237	$2.5\ 10^{-237}$

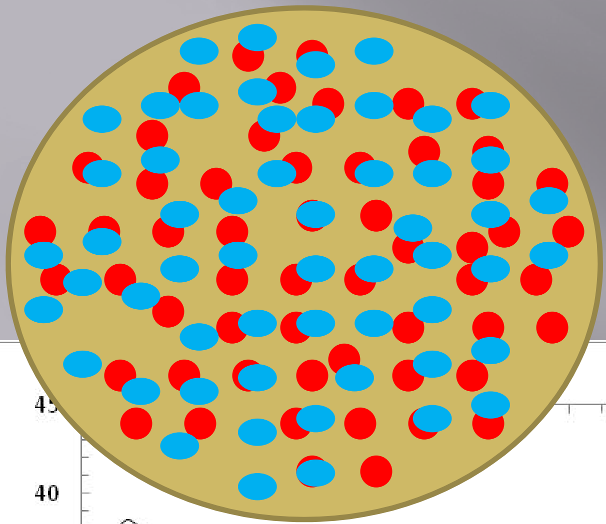
Nuclear reactions in the laboratory

X(a,b)Y

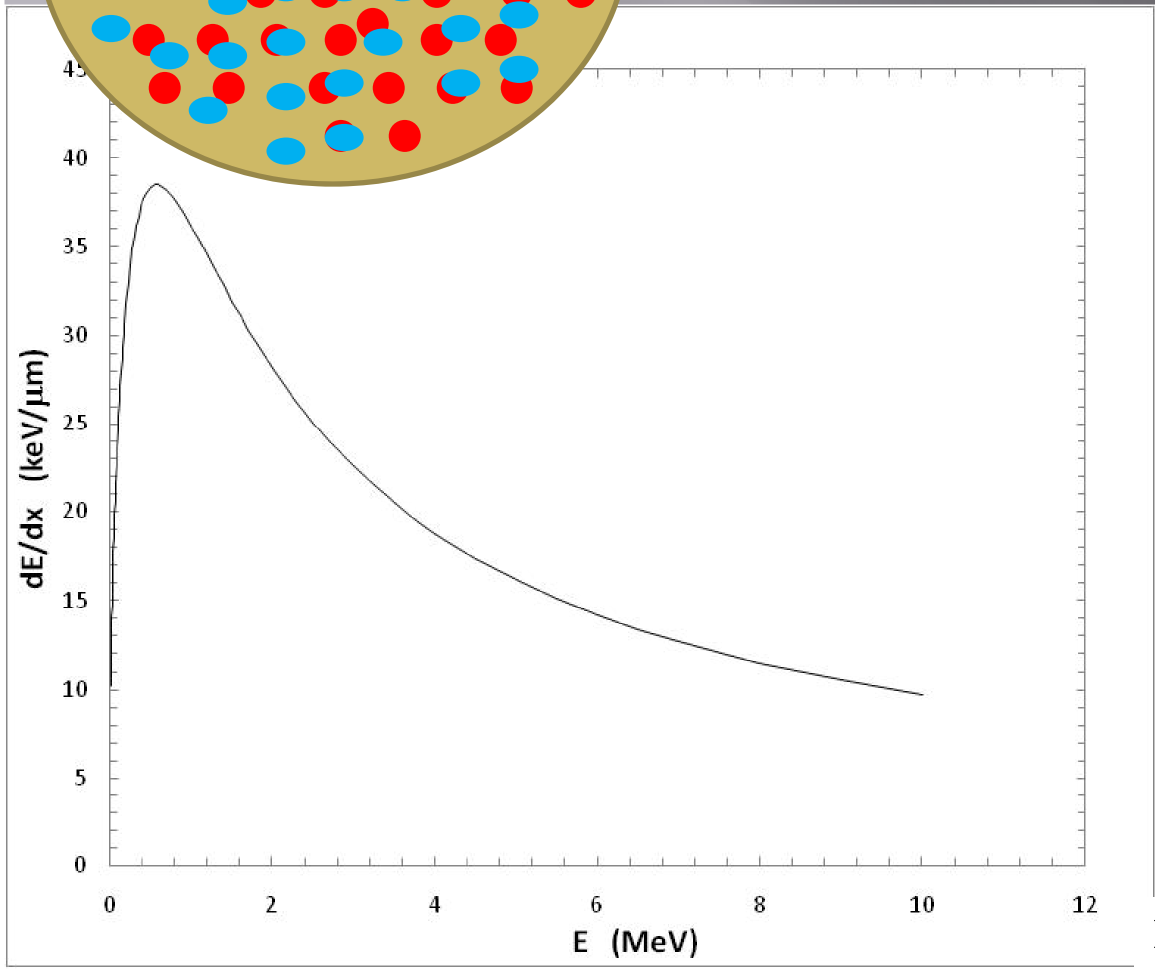


Reaction yield

$$Y = \frac{N_t \sigma}{F} \cdot N_p$$



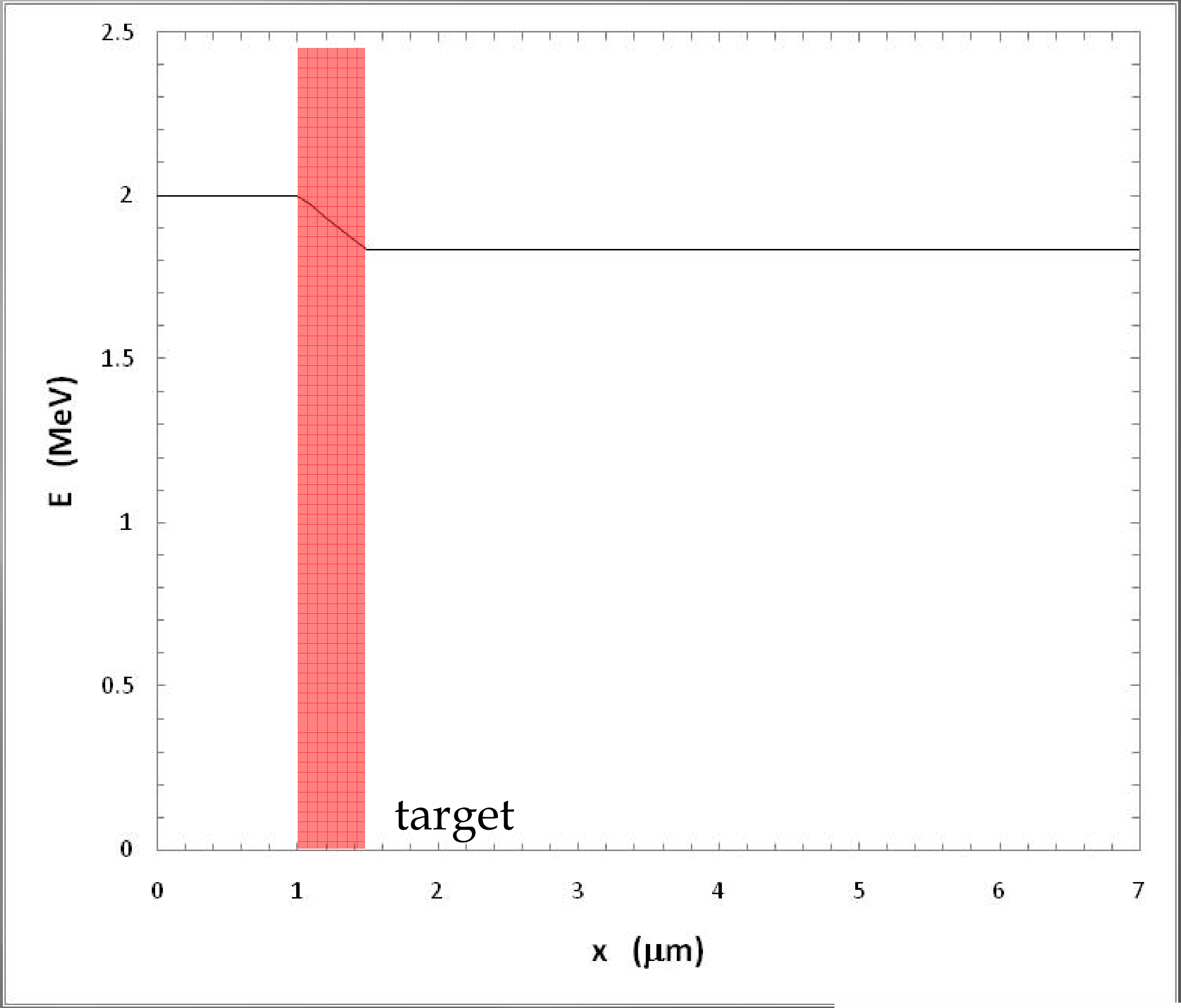
but σ depend on E, and E is not constant through the target



α particles in C
SRIM calulation
www.srim.org

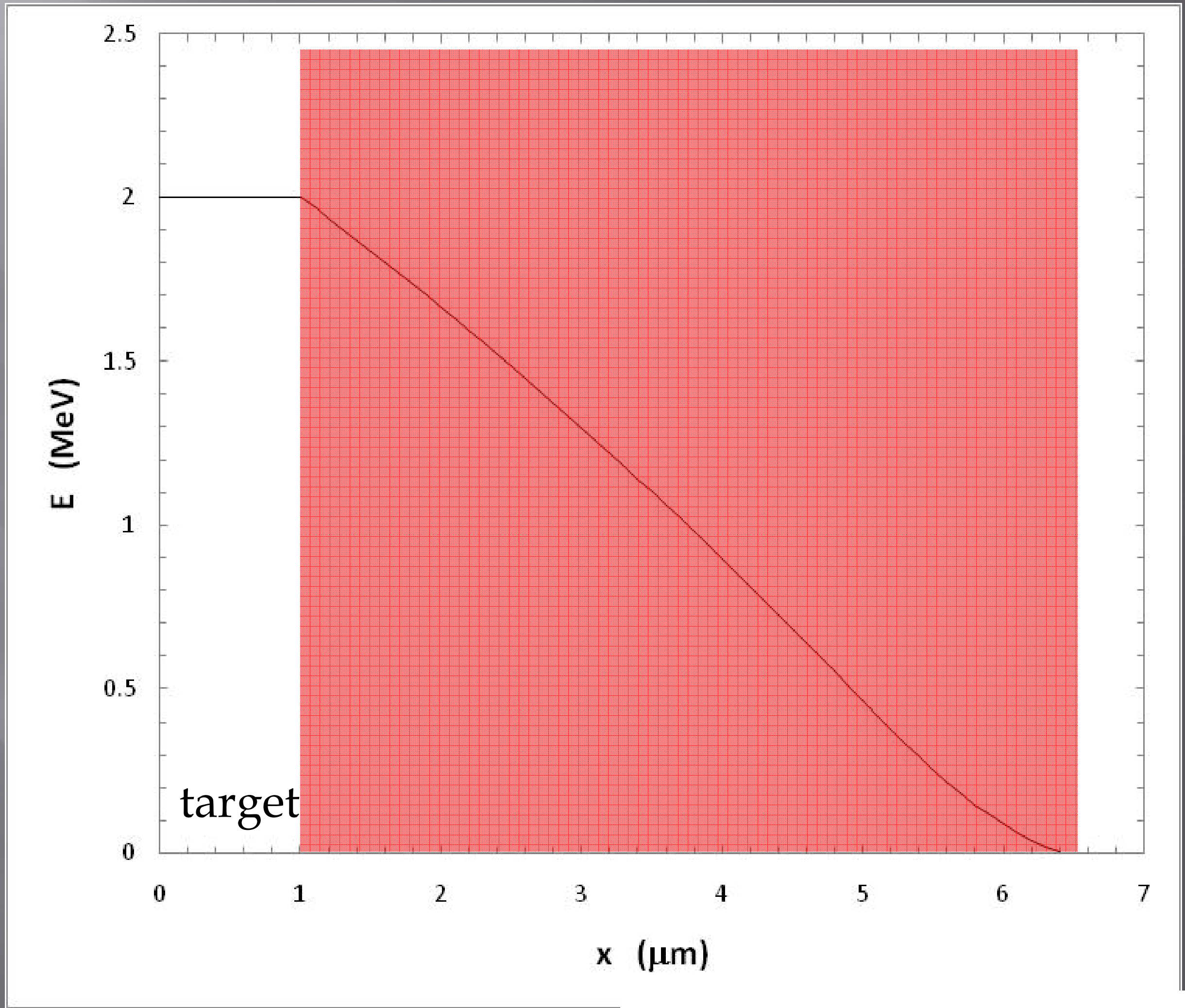
$$E(x) = E_0 - \int_{x_0}^{x} \frac{dE}{dx'}\, dx'$$

α particles in C
SRIM calulation



$$E(x) = E_0 - \int_{x_0}^{x} \frac{dE}{dx'}\, dx'$$

α particles in C
SRIM calulation

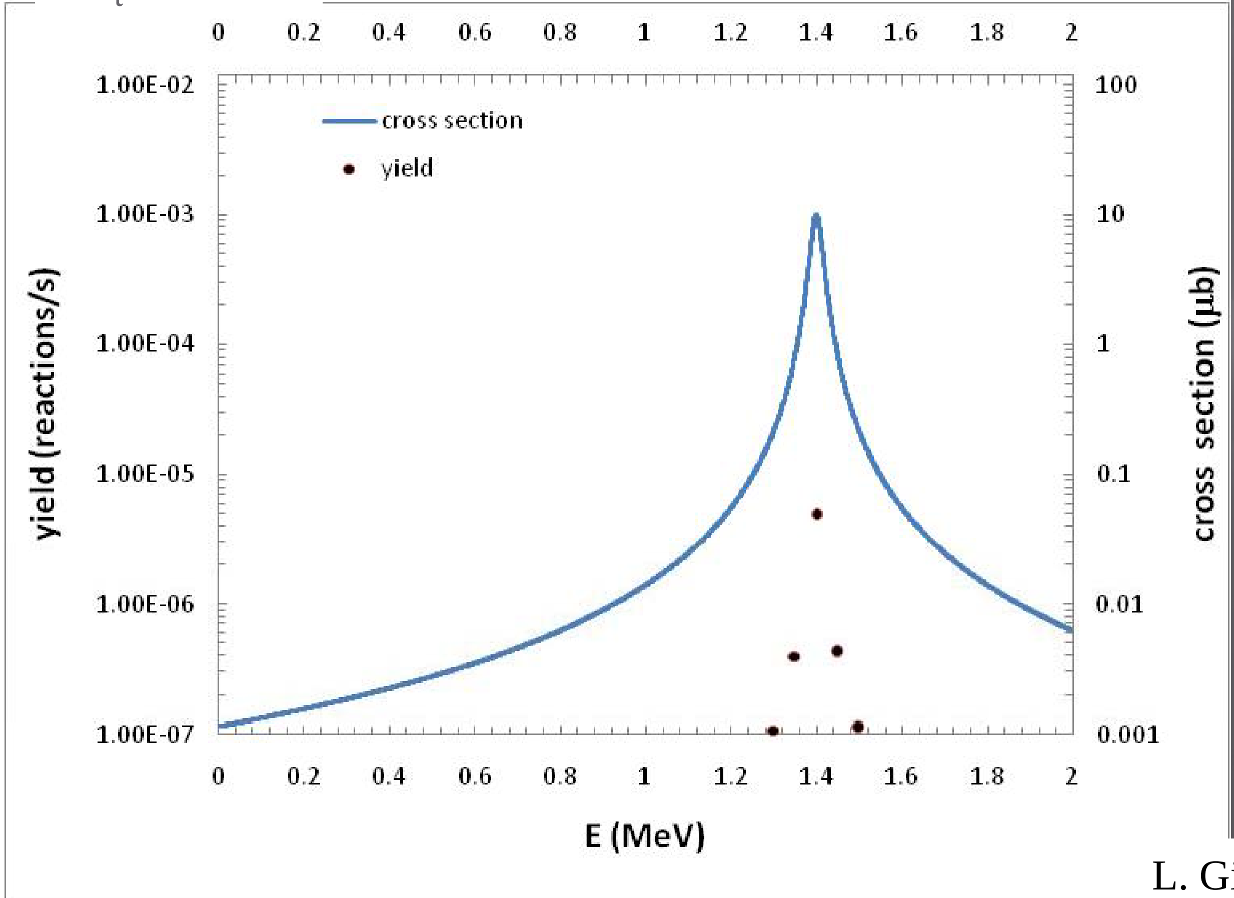


$$Y = \int_{x_0}^{x_1} N_p \frac{n_t \sigma(E)}{F} F dx = \int_{E_1}^{E_0} N_p n_t \sigma(E) \frac{1}{\frac{dE}{dx}} dE$$

Case of a single resonance

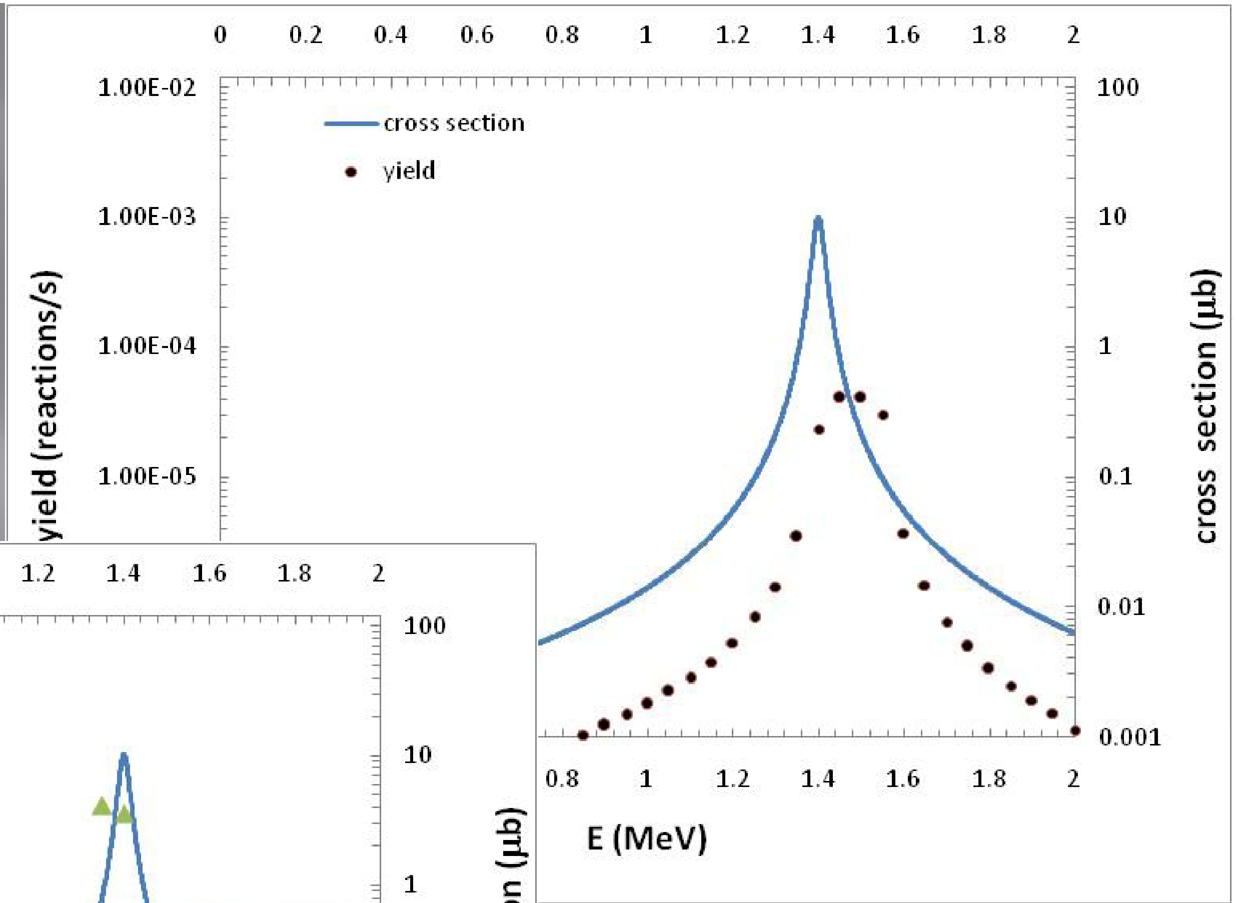
E_r=1.4 MeV
Γ=30 keV

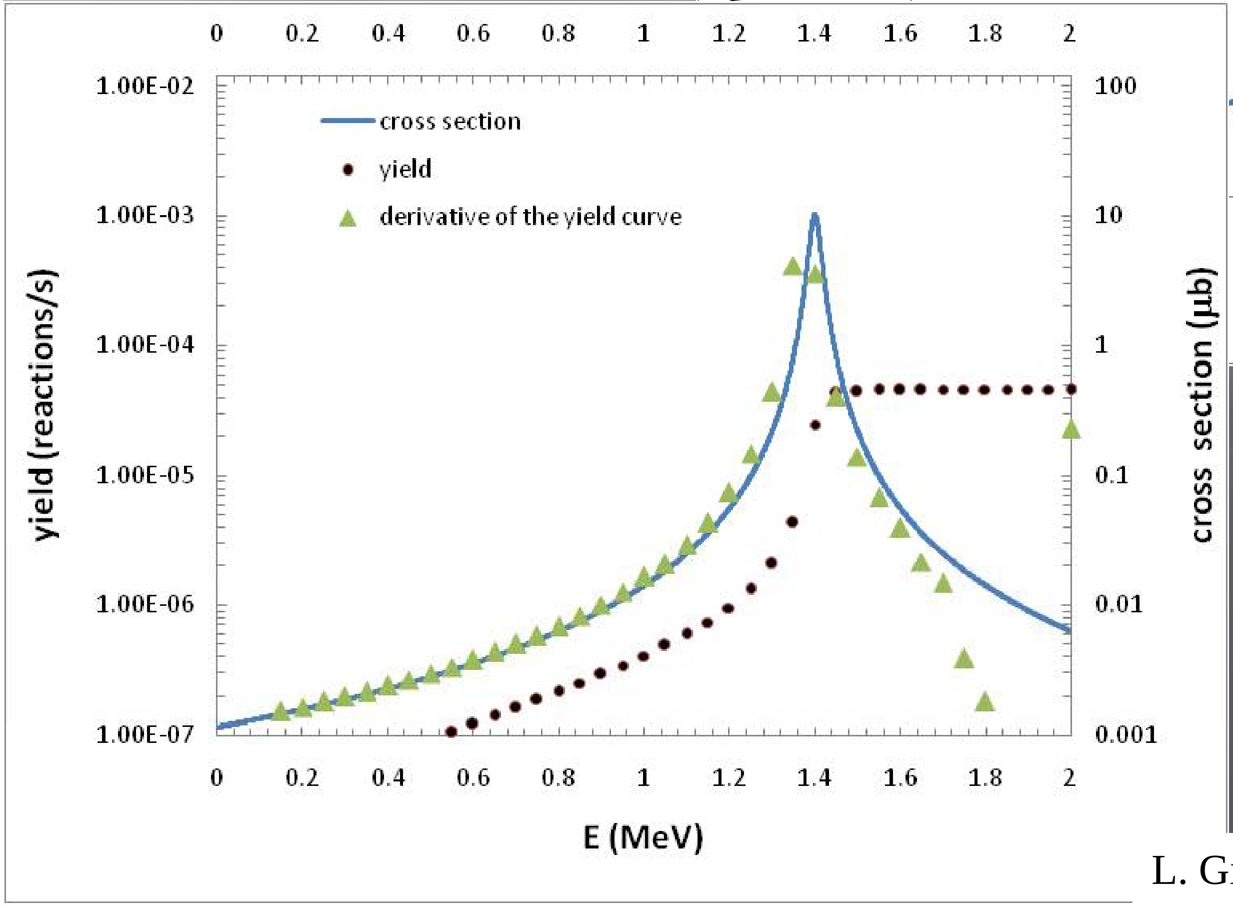
ΔE_t=3 keV



E_r=1.4 MeV
Γ=30 keV

ΔE_t=100 keV





E_r=1.4 MeV
Γ=30 keV

ΔE_t=600 keV
$Y_{thick} \propto \omega\gamma$
$dY_{thick} / dE \propto \sigma$

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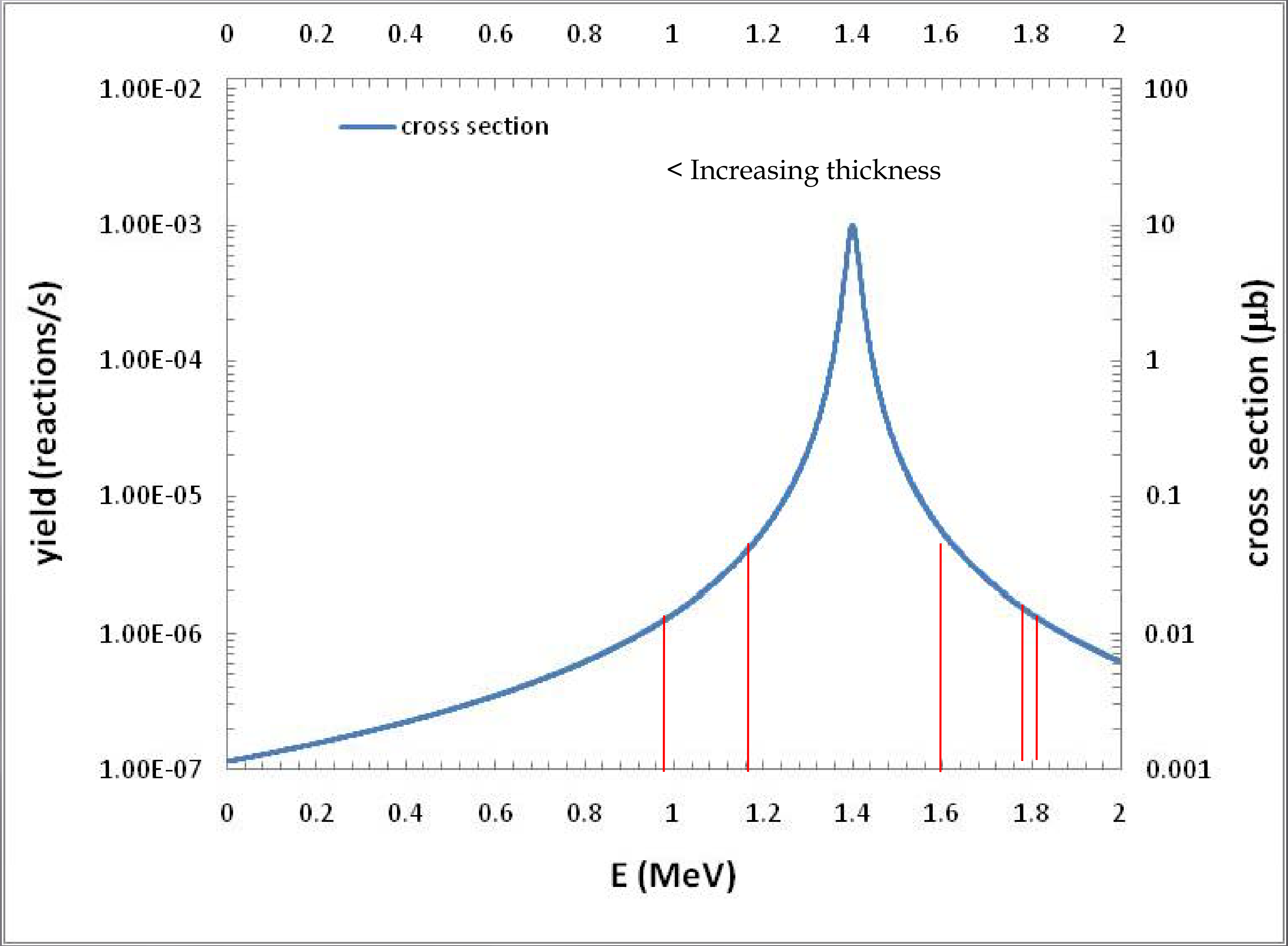


0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2
1.00E-02
1.00E-03
1.00E-04
1.00E-05
1.00E-06
1.00E-07
100
10
1
0.1
0.01
0.001
cross section
< Increasing thickness
yield (reactions/s)
cross section (µb)
E (MeV)

Some remarks:

1- Avoid intermediate thicknesses
(if you know the width and if you can prepare appropriate targets)

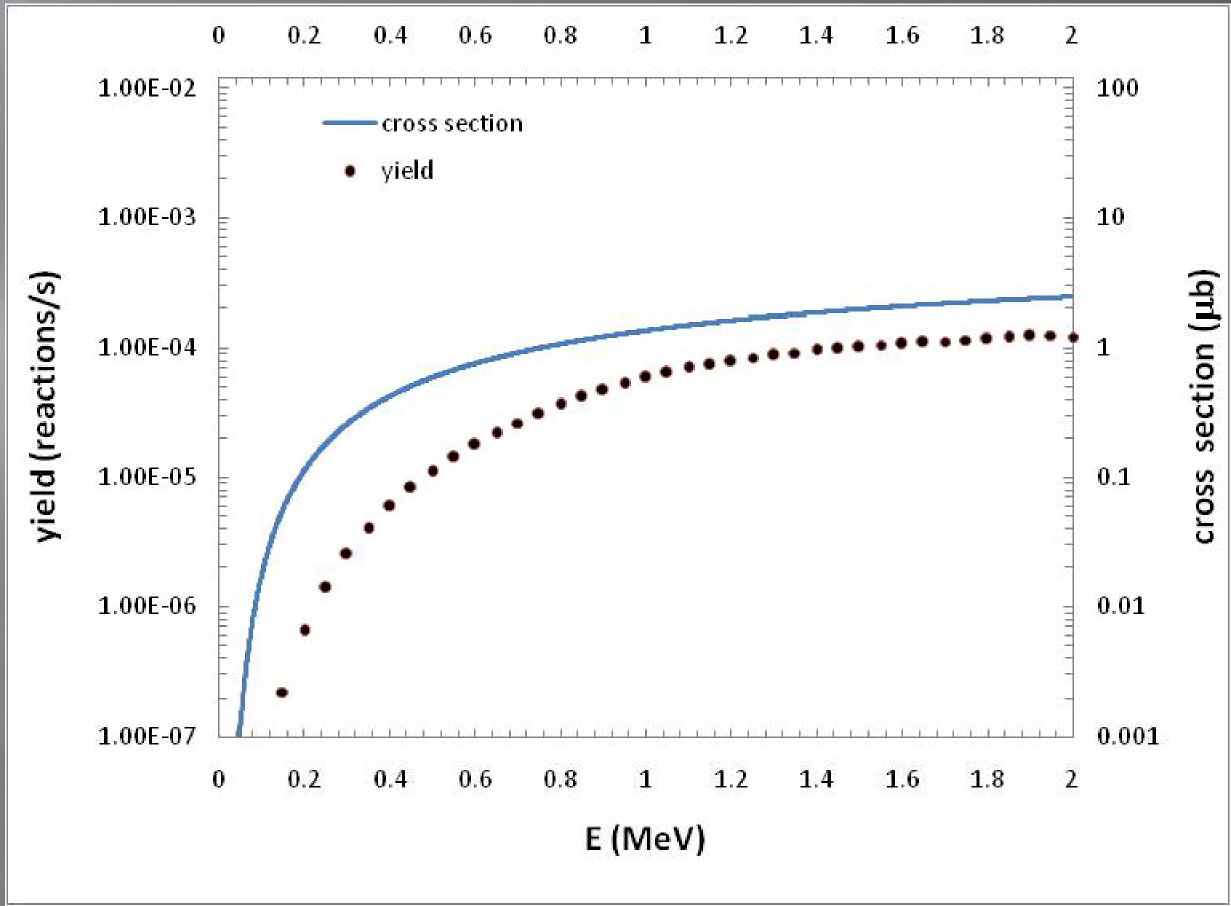
2- Be aware that the derivated thick target yield will be affected
by large uncertainties for weak resonances

3- Beam energy spread complicates things

4- We plotted yields as a function of the beam energy: that’s not good

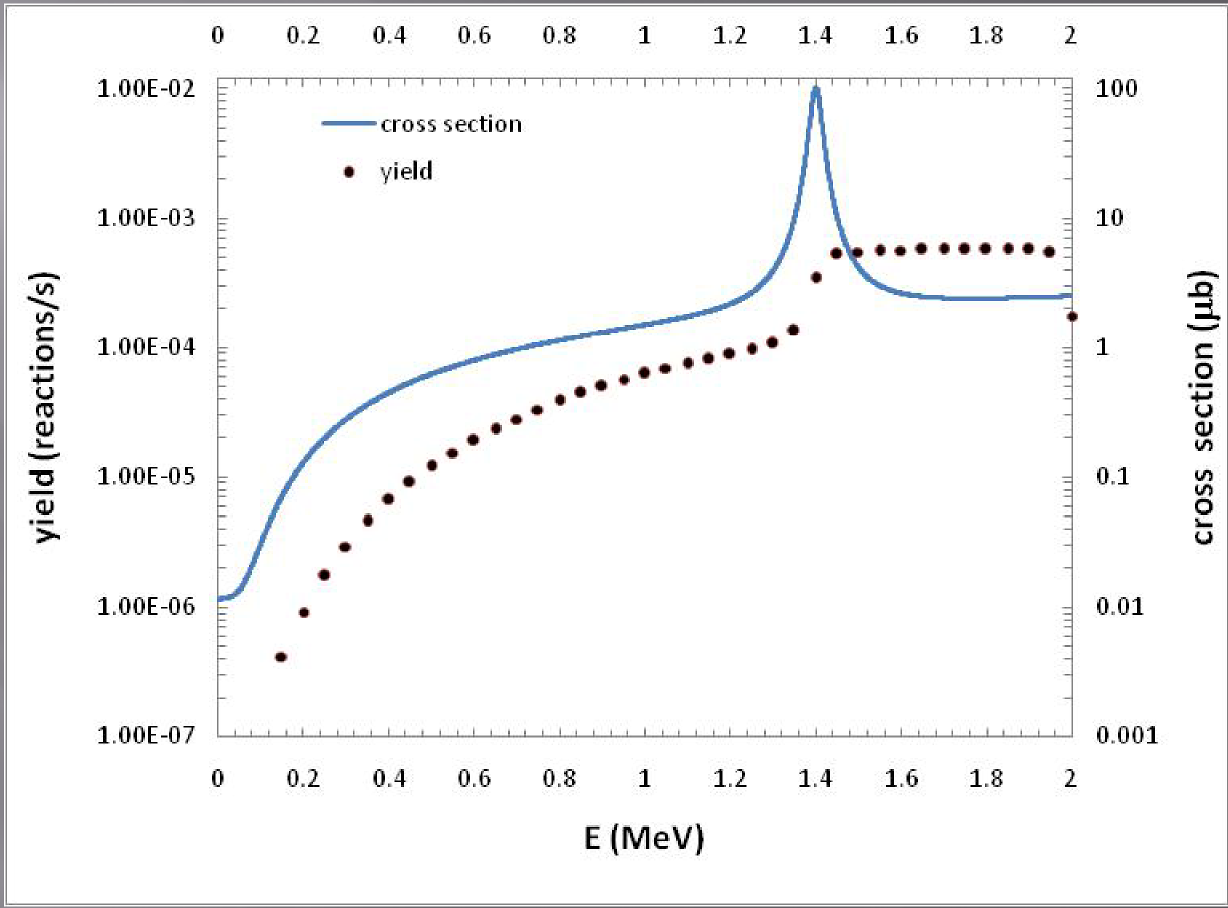
5-Note the reaction yield is not the counting rate in the detector

Non resonant case



E_{eff}

resonant+not resonant



Effective energy for the cs extracted from a yield

1

$$\int_{E_0-\Delta E}^{E_{eff}} \sigma(E)dE = \frac{1}{2}\int_{E_0-\Delta E}^{E_0} \sigma(E)dE$$

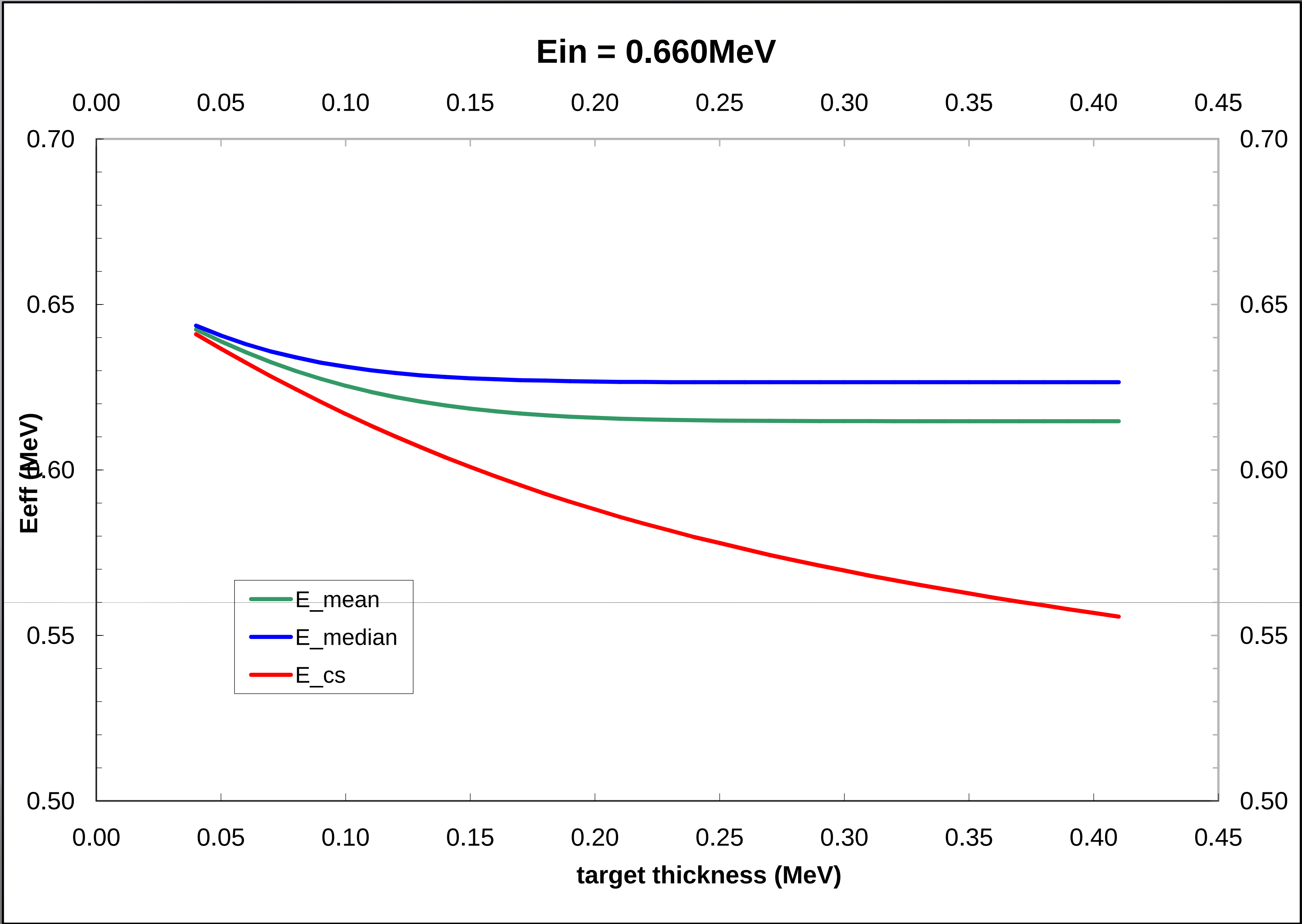
2

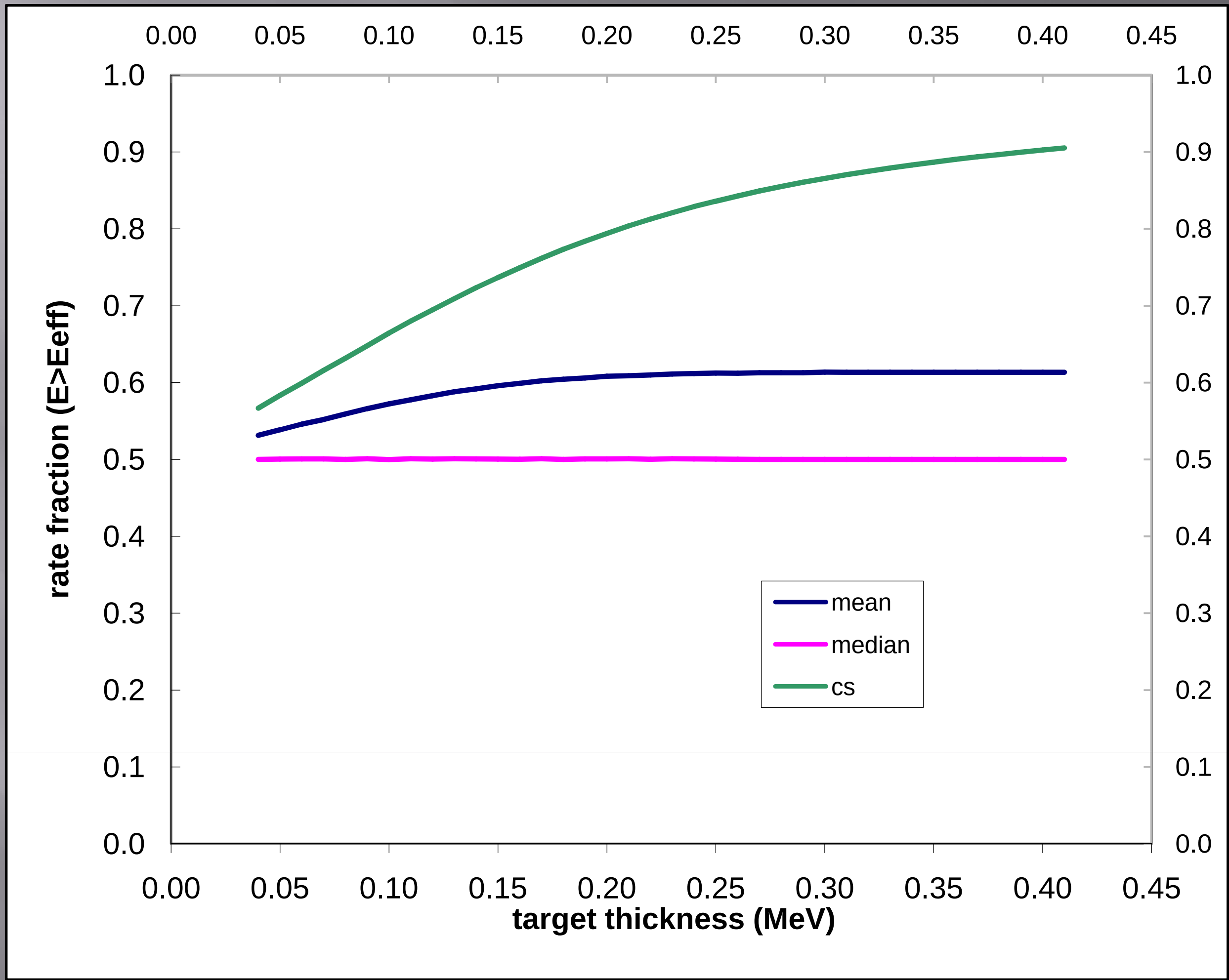
$$E_{eff} = \frac{\int_{E_0-\Delta E}^{E_0} \sigma(E)EdE}{\int_{E_0-\Delta E}^{E_0} \sigma(E)dE}$$

3

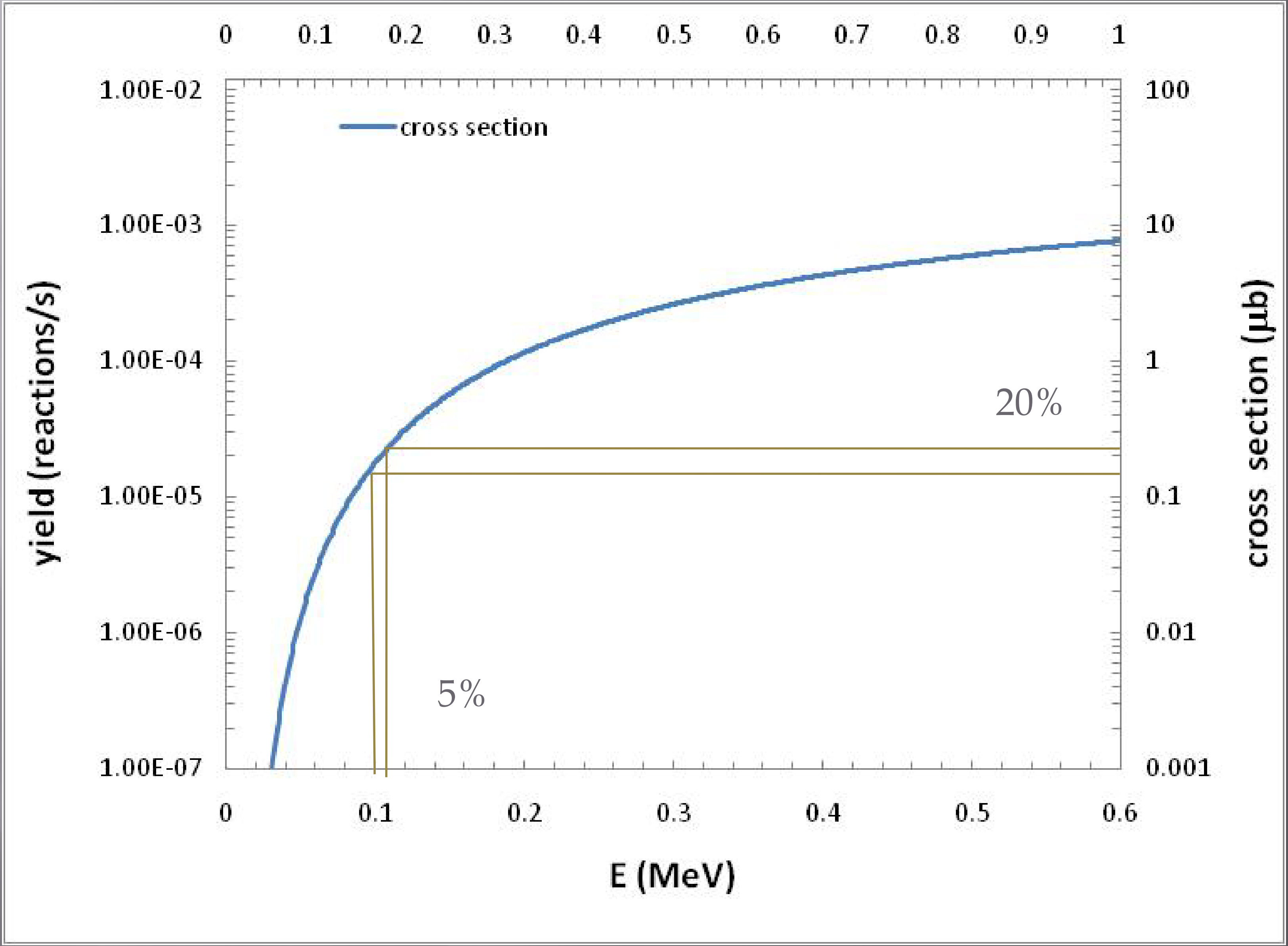
$$\frac{N_r}{N_p N_t} = \frac{\int_{E_0-\Delta E}^{E_0} \sigma(E)dE}{\Delta E} = \sigma_{eff}$$

$$\sigma(E_{eff}) = \sigma_{eff}$$

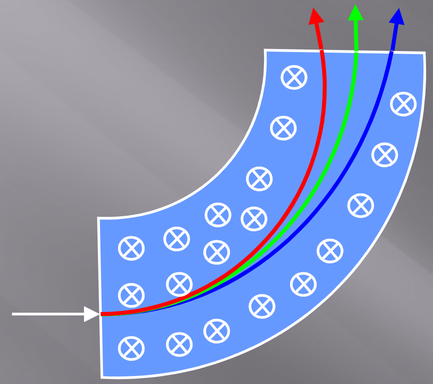

Ein = 0.660MeV
0.00 0.05 0.10 0.15 0.20 0.25 0.30 0.35 0.40 0.45
0.70 0.65 0.60 0.55 0.50
Eeff (MeV)
target thickness (MeV)
E_mean
E_median
E_cs



rate fraction (E>Eeff)
target thickness (MeV)
mean
median
cs
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35
0.40
0.45
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0


0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.00E-02
1.00E-03
1.00E-04
1.00E-05
1.00E-06
1.00E-07
100
10
1
0.1
0.01
0.001
cross section
20%
5%
yield (reactions/s)
cross section (µb)
0
0.1
0.2
0.3
0.4
0.5
0.6
E (MeV)

Energy calibration

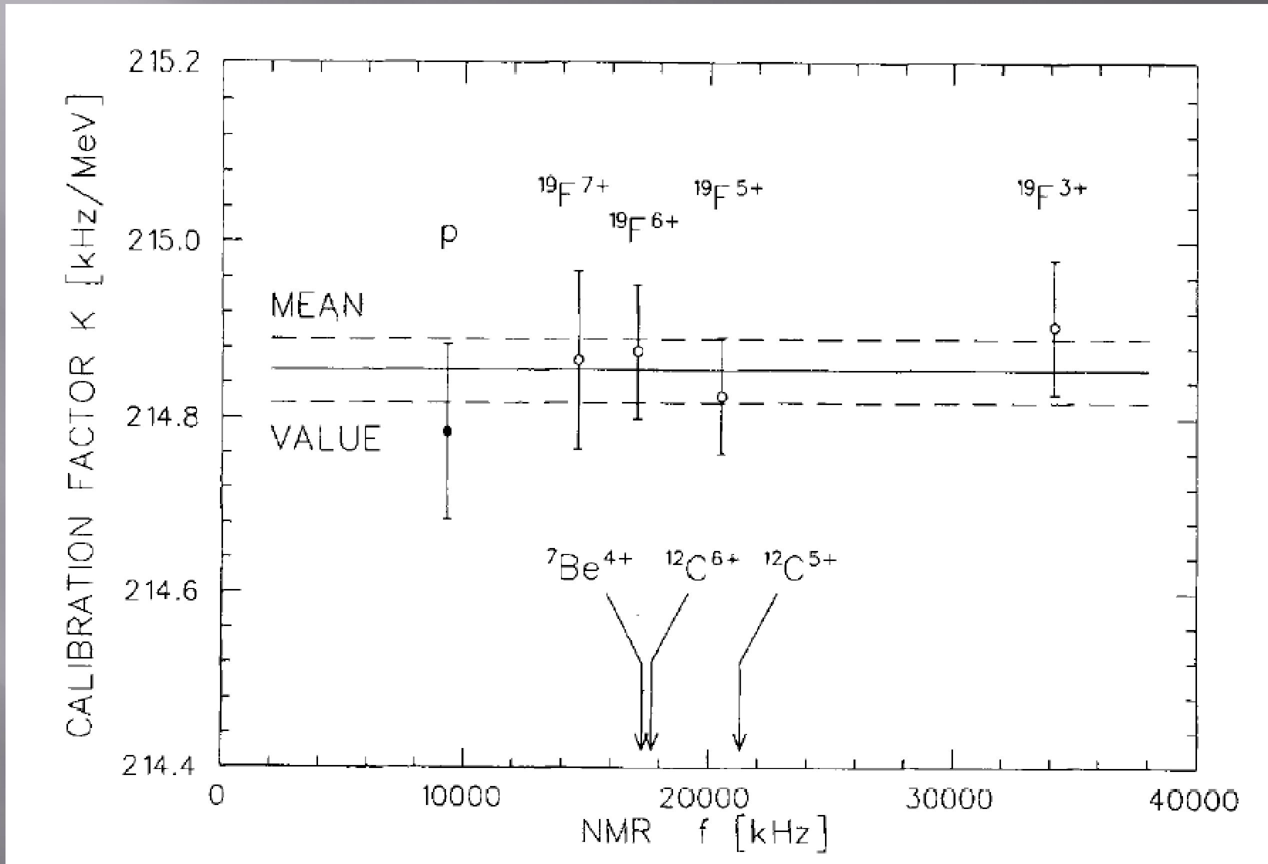


magnetic dipole

$$BR = \frac{(2mc^2E + E^2)^{1/2}}{qc} \approx p/q$$

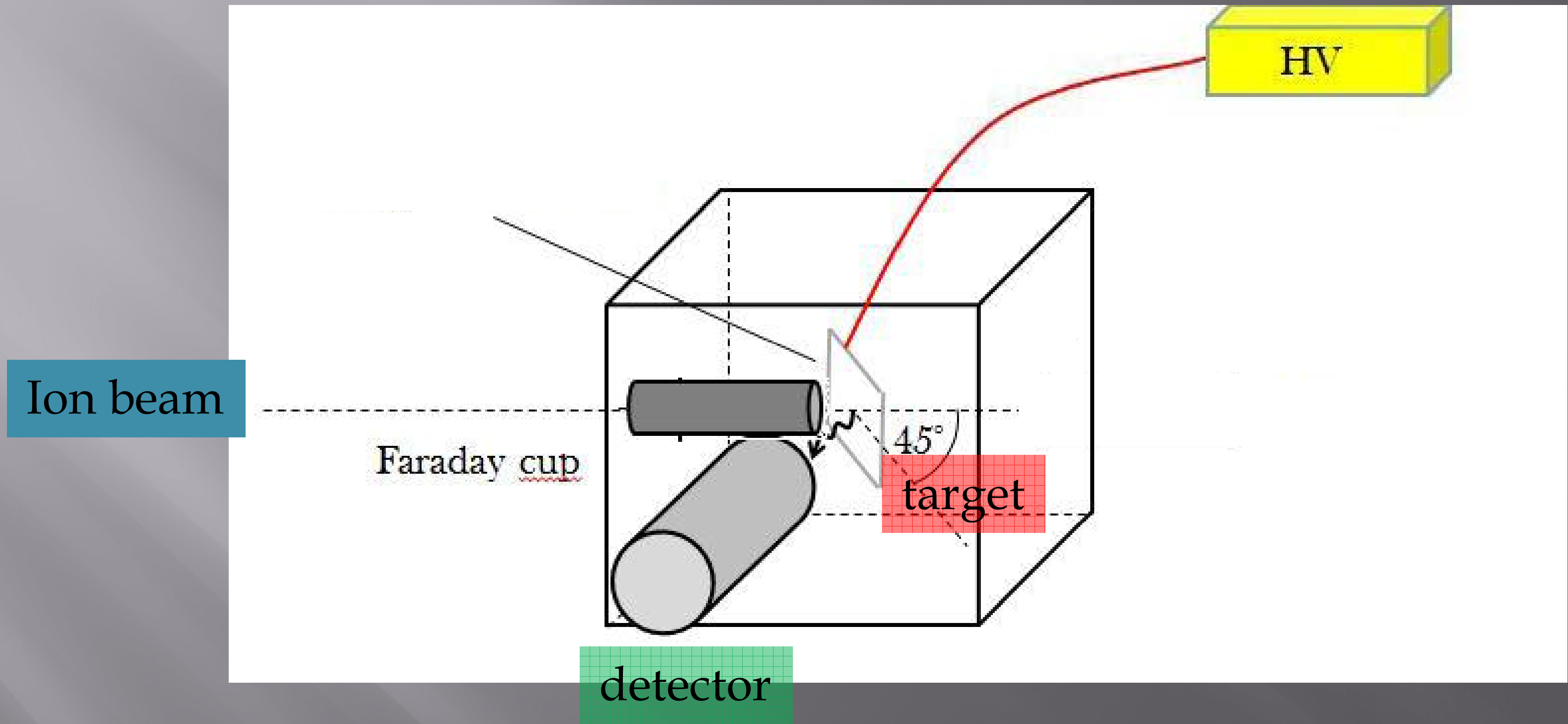
$$K = qB/(2mc^2E + E^2)^{1/2}$$

Courtesy D. Schuermann

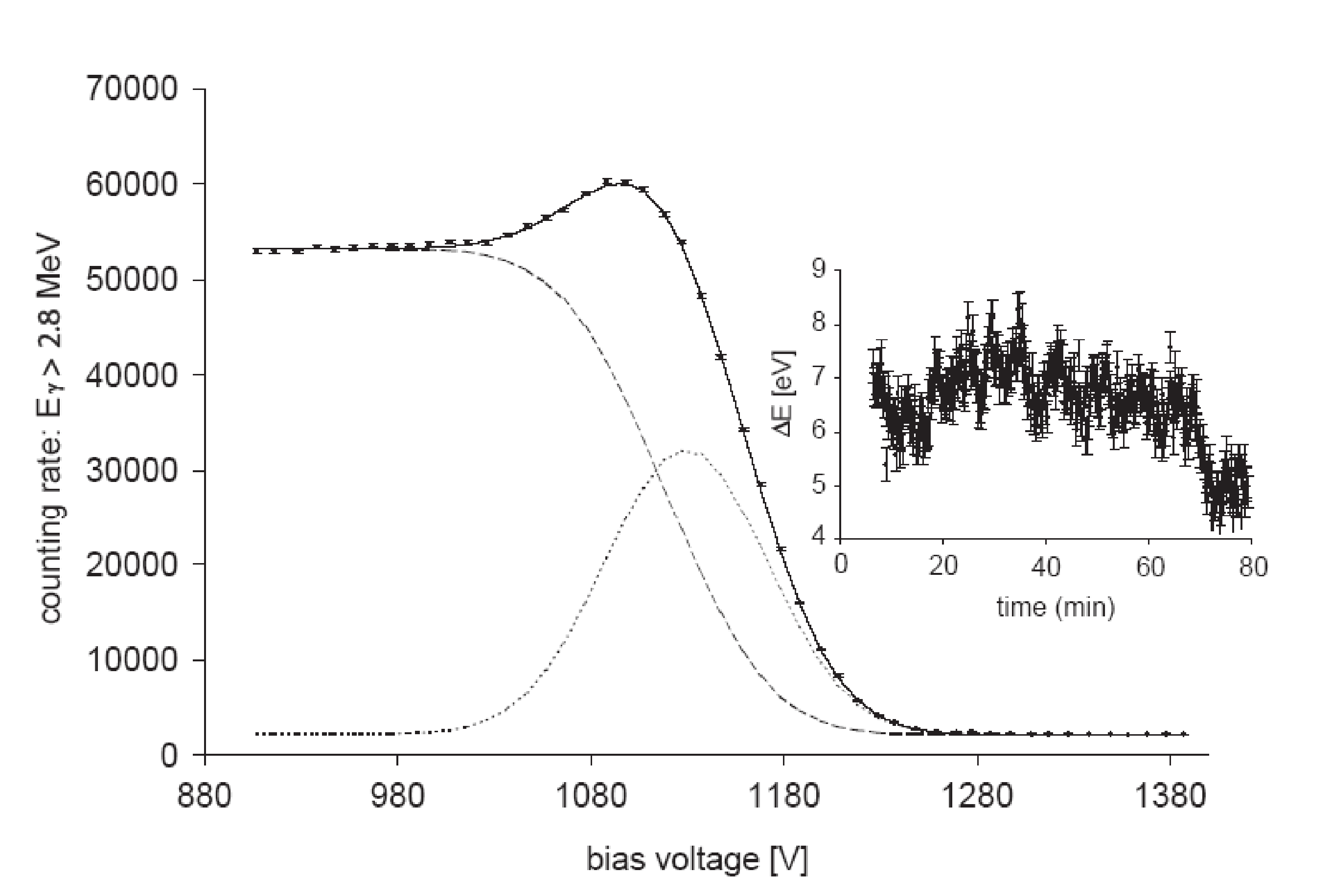


Gialanella et al. NIM A 376 (1996) 174-184

Energy calibration and energy spread



$^{25}Mg(p,\gamma)^{26}Al \;\; at \; E_p = 389.24 \pm 0.11 \, keV$



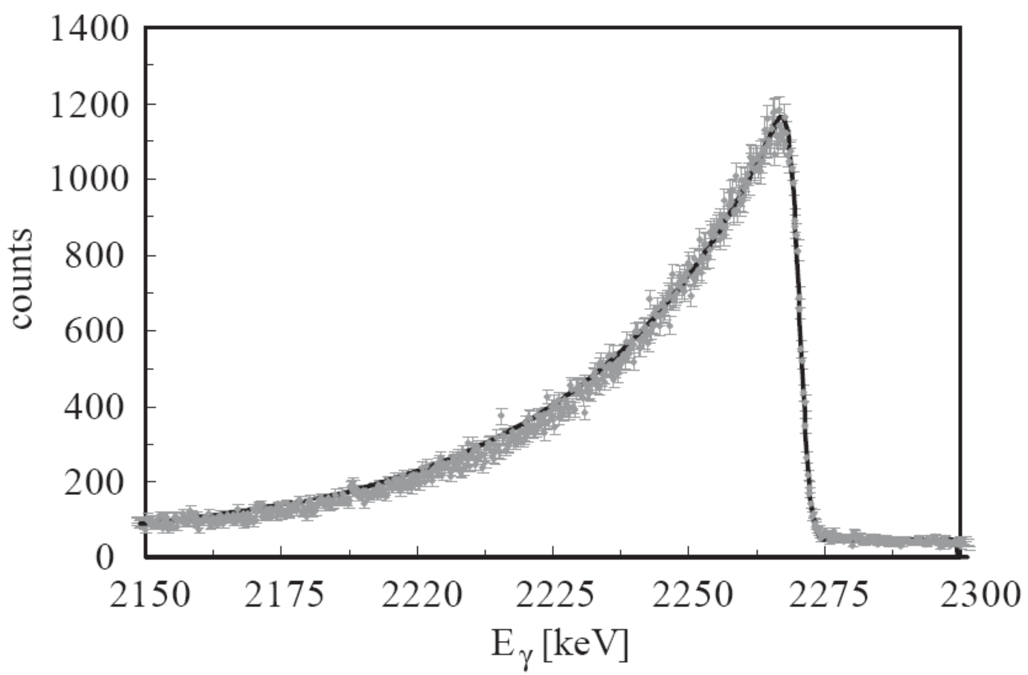
Formicola et al NIM A 507 (2003) 609–616

Lewis Phys. Rev. 125 (1962)

Table 1
Resonance parameters

Reaction	Resonance energy E_R (keV)	Resonance width Γ (eV)	Doppler broadening ΔE_D (eV)	Beam spread ΔE_B (eV)	HV + PV (kV)	Shift (keV)
$^{23}Na(p,\gamma)^{24}Mg$	308.75 ± 0.06[a]	<36[a]	58[a]	71	311.24	2.49 ± 0.06
$^{25}Mg(p,\gamma)^{26}Al$	316.11 ± 0.11[b]	<37[a]	58[a]	120	318.83	2.65 ± 0.11
$^{26}Mg(p,\gamma)^{27}Al$	338.30 ± 0.10[c]	<40[a]	59[a]	101	340.80	2.50 ± 0.10
$^{25}Mg(p,\gamma)^{26}Al$	389.24 ± 0.11[b]	<4[d]	62[a]	72	392.17	2.93 ± 0.11

$^{13}C(p,\gamma)^{14}N \quad E_p = 100 - 400\ keV$



Experimental determination of reaction cross sections

Direct methods:

- very low cross sections -> low counting rates
- measure outside Gamow window -> extrapolation (reaction mechanism)
- cosmic radiation + nat. roombckg -> background
- Beam/target induced bckg -> background

<u>key improvements:</u>

- Targets
- Detectors

To do

- Targets: solid and gas targets
- Detectors: gamma rays and charged particles