Lecture #1: Nuclear and Thermonuclear Reactions

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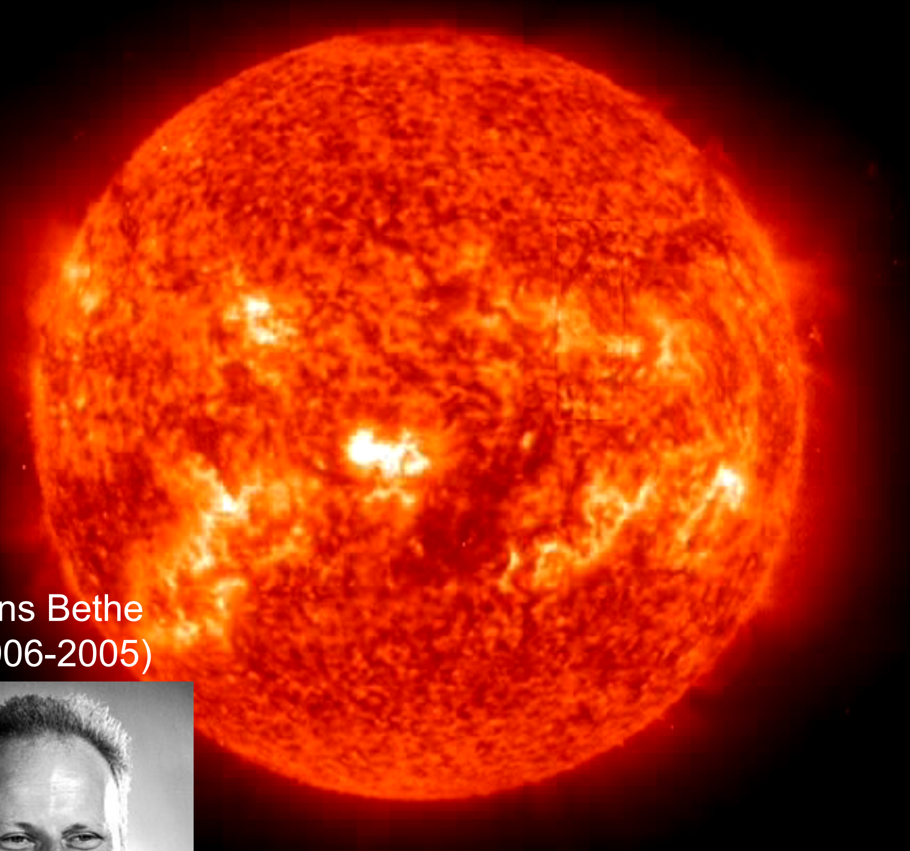
TUNL

Introduction

SOlar and Heliospheric Observatory (SOHO)



Hans Bethe
(1906-2005)

Life on Earth depends on nuclear processes deep inside the Sun

Fusion of H to He:

Bethe & Critchfield (1938)
[pp chains]
Bethe 1939; von Weizsäcker 1938
[CNO cycle]

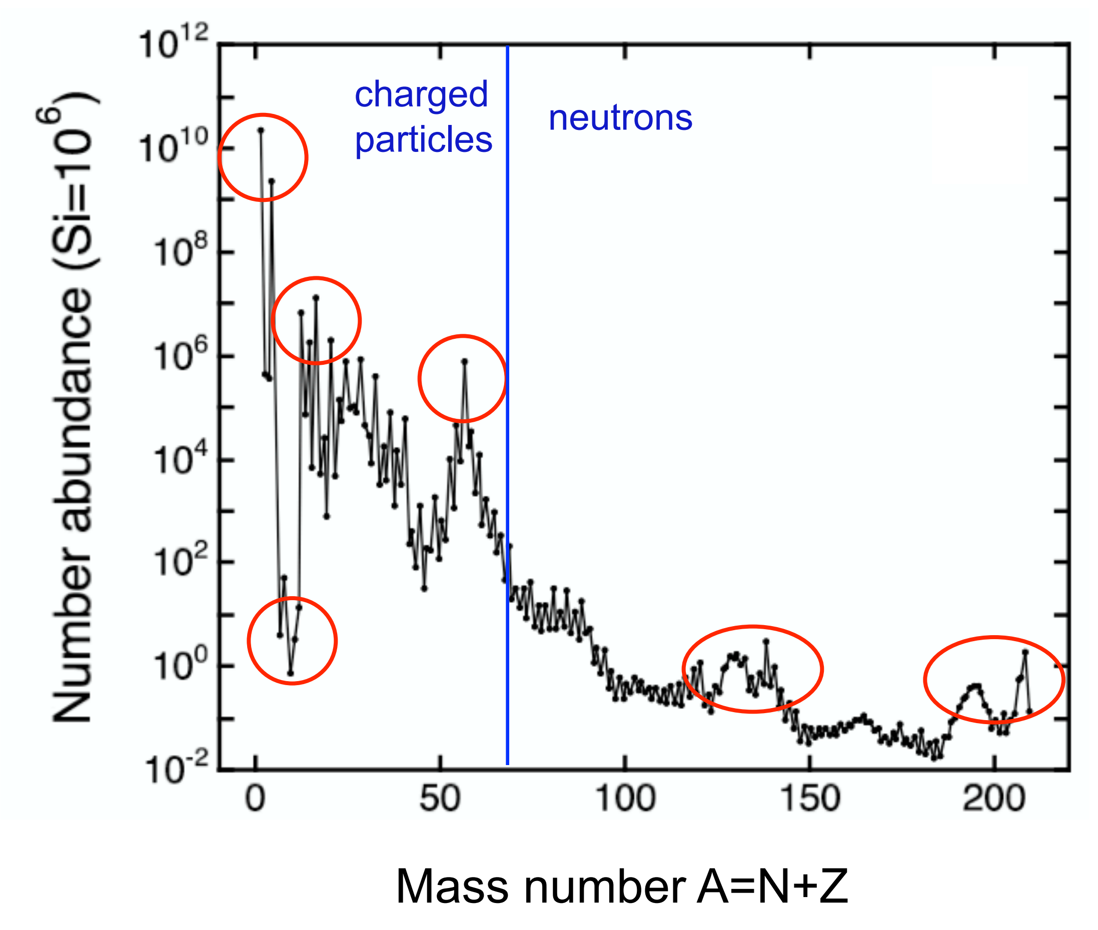
Nobel prize to Hans Bethe (1967)

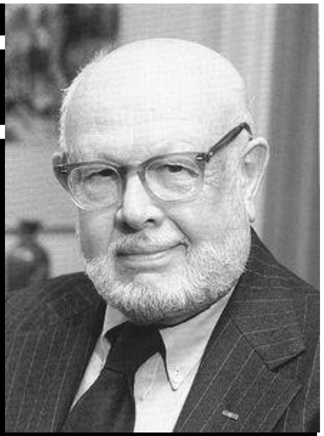
Accurate nuclear physics information is crucial for understanding of stars

How do other stars produce energy? How do they evolve?

Sun did not produce elements found on Earth…

Solar system abundances





Willy Fowler (1911-95)

- Suess & Urey, Rev. Mod. Phys. 28, 53 (1956)
- Lodders, Palme & Gail, Landolt-Boernstein New Series VI/4B (Springer 2009)

Foundation of modern theory of nuclear astrophysics:

- Burbidge, Burbidge, Fowler and Hoyle (1957)
- Cameron (1957)

Nobel prize to Willy Fowler (1983)

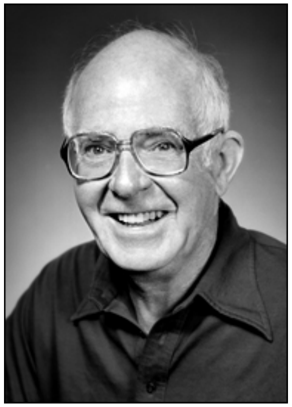
Direct evidence for stellar nucleosynthesis

(i) Solar neutrinos

- first and only *direct* empirical evidence of how Sun generates energy was performed by detecting solar neutrinos [mostly from ^{8}B decay] at the Homestake gold mine, South Dakota, USA

- disagreement of predicted and measured neutrino flux: "solar neutrino problem" [giving later rise to discovery of neutrino oscillations]

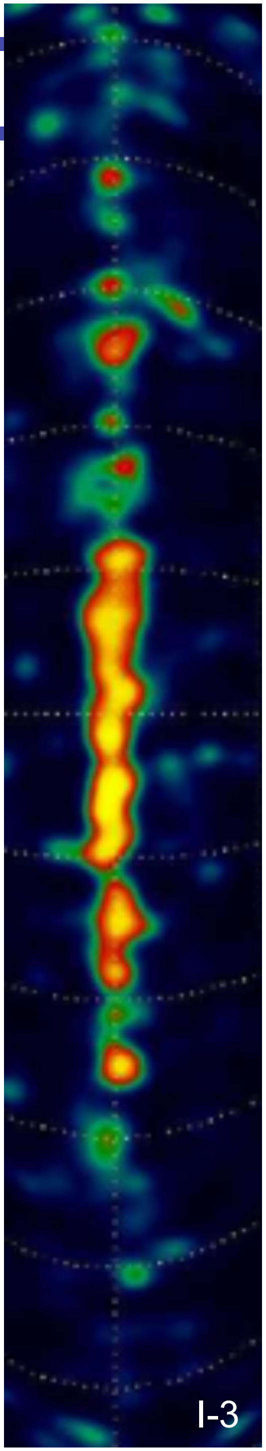
- Nobel prize to Ray Davis (2002)



Ray Davis
(1914-2006)

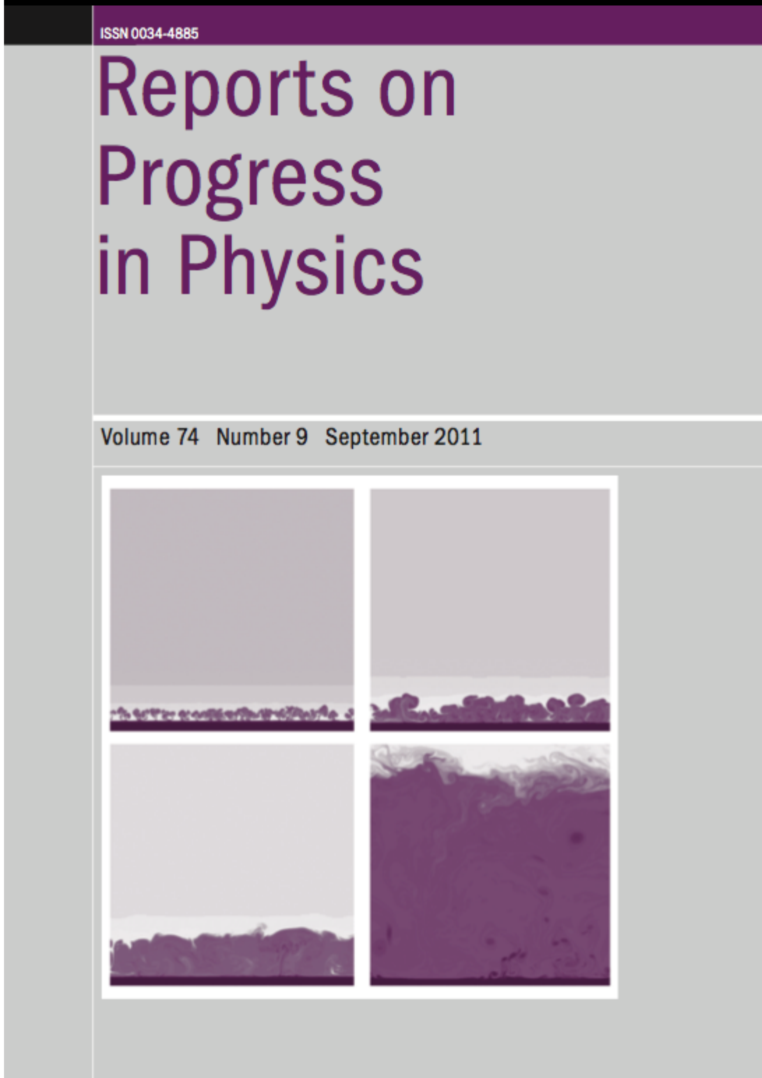
(ii) γ-ray astronomy

- radioactive ('live") ^{26}Al has bee observed in the Galaxy [see image on right]
- $T_{1/2}(^{26}Al)$=720,000 years; time scale of Galactic chemical evolution: 10^9 years
- from photon intensity: 1-2 solar masses of ^{26}Al in Galaxy
- conclusion: nucleosynthesis is ongoing



COMPTEL map of 1.8 MeV photon intensity

Recent review article



(i) Why do predictions of helioseismology disagree with those of the standard solar model?

(ii) What is the solution to the lithium problem in Big Bang nucleosynthesis?

(iii) What do the observed light-nuclide and s-process abundances tell us about convection and dredge-up in massive stars and AGB stars?

(iv) What are the production sites of the γ-ray emitting radioisotopes ^{26}Al, ^{44}Ti and ^{60}Fe?

(v) What is the origin of about 30 rare and neutron-deficient nuclides beyond the iron peak (p-nuclides)?

(vi) What causes core-collapse supernovae to explode?

(vii) What is the extent of neutrino-induced nucleosynthesis (ν-process)?

(viii) What is the extent of the nucleosynthesis in proton-rich outflows in the early ejecta of core-collapse supernovae (νp-process)?

(ix) What are the sites of the r-process?

(x) What causes the discrepancy between models and observations regarding the mass ejected during classical nova outbursts?

(xi) Which are the physical mechanisms driving convective mixing in novae?

(xii) What are the progenitors of type Ia supernovae?

(xiii) What is the nucleosynthesis endpoint in type I x-ray bursts? Is there any matter ejected from those systems?

(xiv) What is the impact of stellar mergers on Galactic chemical abundances?

(xv) What are the production and acceleration sites of Galactic cosmic rays?

Jose & Iliadis, “Nuclear Astrophysics: the Unfinished Quest for the Origin of the Elements”, Reports on Progress in Physics 74, 096901 (2011)

Nuclear reactions

Definition of cross section:

$$\sigma \equiv \frac{(\text{number of interactions per time})}{(\text{number of incident particles per area per time})\,(\text{number of target nuclei within the beam})} = \frac{N_r}{N_0 N_t}$$

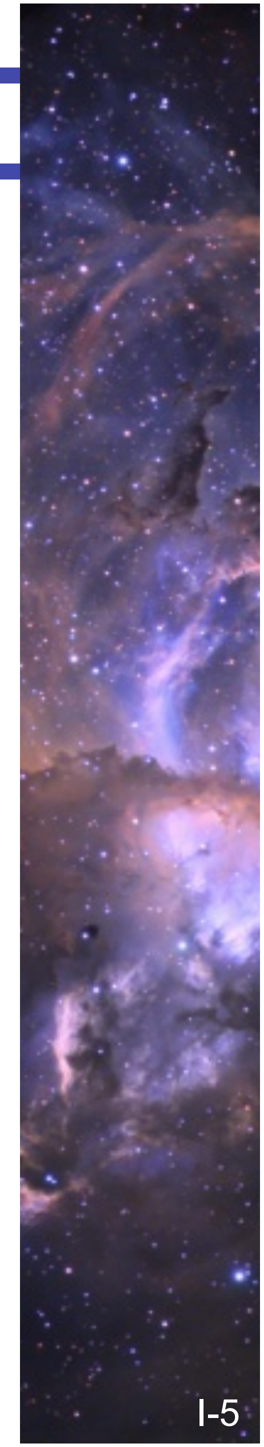
Unit: 1 barn=10^{-28} m^2

Example: $^1H + {}^1H \rightarrow {}^2H + e^+ + \nu$ (first step of pp chain)

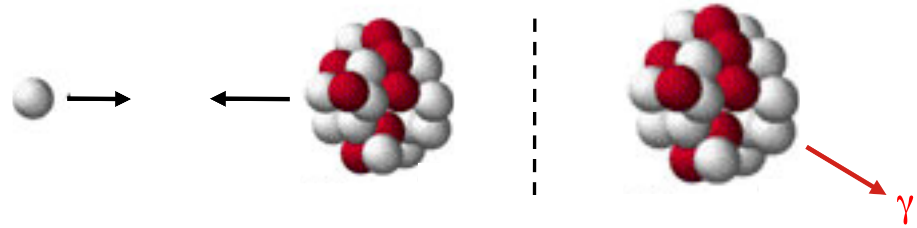
$\sigma_{theo}=8\times10^{-48}$ cm^2 at E_{lab}=1 MeV [E_{cm}=0.5 MeV]

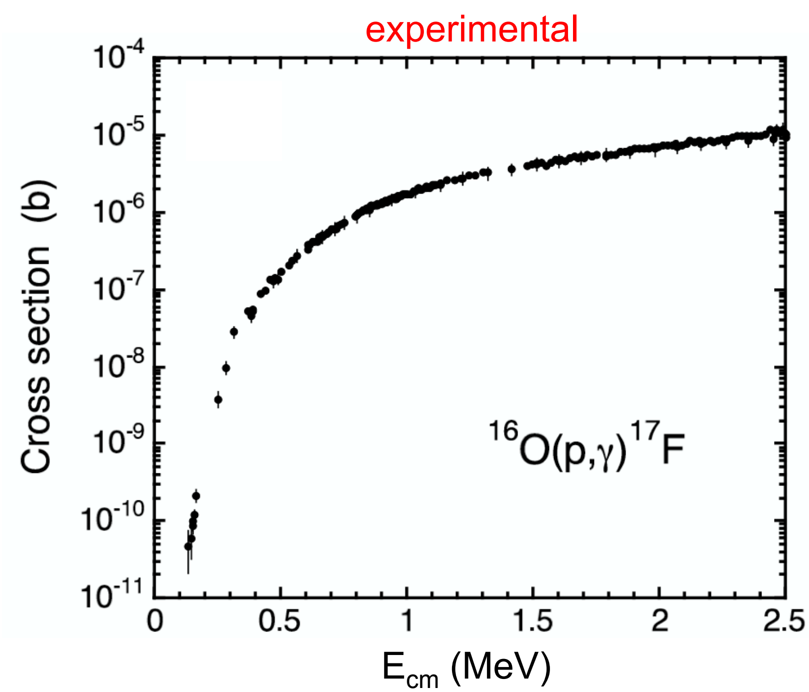
1 ampere (A) proton beam (6×10^{18} p/s) on dense proton target (10^{20} p/cm^2)

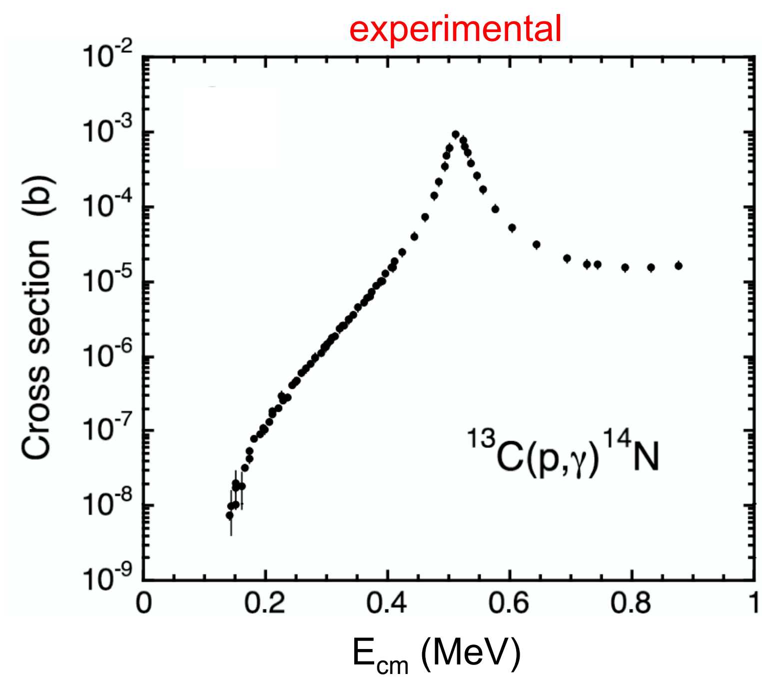
gives only 1 reaction in 6 years of measurement!



Cross sections

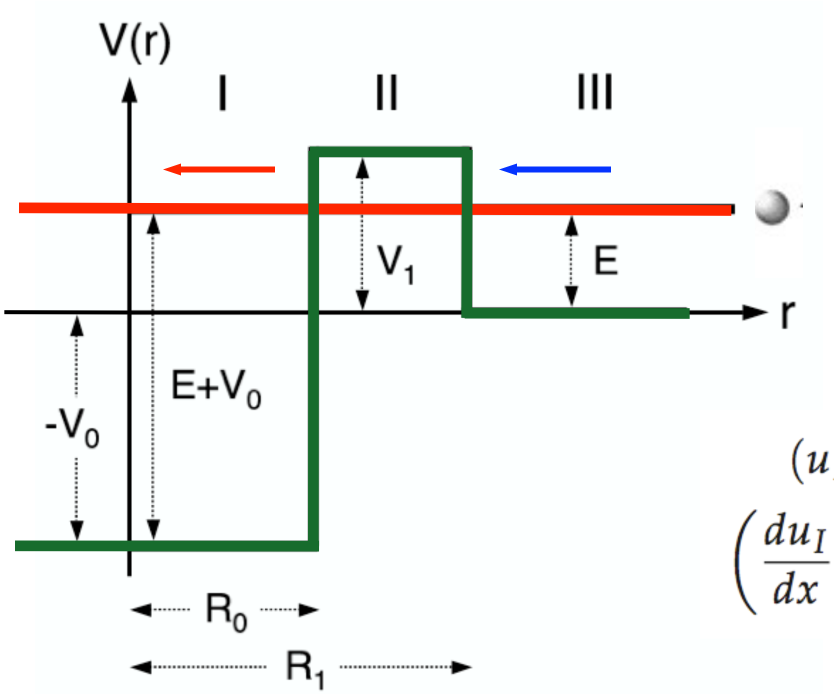






(i) why does the cross section fall drastically at low energies?
(ii) where is the peak in the cross section coming from?

A simple example in 1 dimension



Wave function solutions:

$$u_I = Ae^{iKx} + Be^{-iKx} \qquad K^2 = \frac{2m}{\hbar^2}(E+V_0)$$

$$u_{II} = Ce^{-\kappa x} + De^{\kappa x} \qquad \kappa^2 = \frac{2m}{\hbar^2}(V_1 - E)$$

$$u_{III} = Fe^{ikx} + Ge^{-ikx} \qquad k^2 = \frac{2m}{\hbar^2}E$$

Continuity condition:

$$(u_I)_{R_0} = (u_{II})_{R_0} \qquad (u_{II})_{R_1} = (u_{III})_{R_1}$$

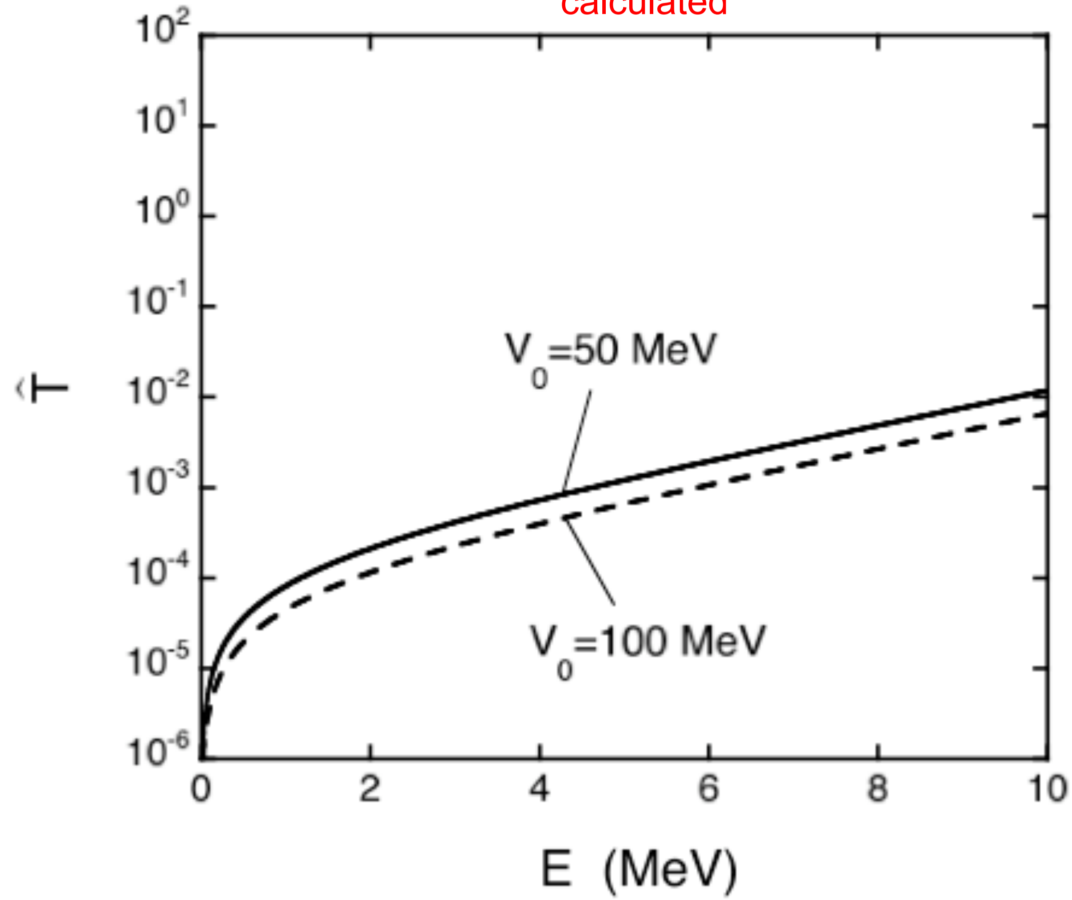
$$\left(\frac{du_I}{dx}\right)_{R_0} = \left(\frac{du_{II}}{dx}\right)_{R_0} \qquad \left(\frac{du_{II}}{dx}\right)_{R_1} = \left(\frac{du_{III}}{dx}\right)_{R_1}$$

Transmission coefficient: $\hat{T} = \frac{K}{k}\frac{|B|^2}{|G|^2} \approx e^{-(2/\hbar)\sqrt{2m(V_1-E)}(R_1-R_0)}$

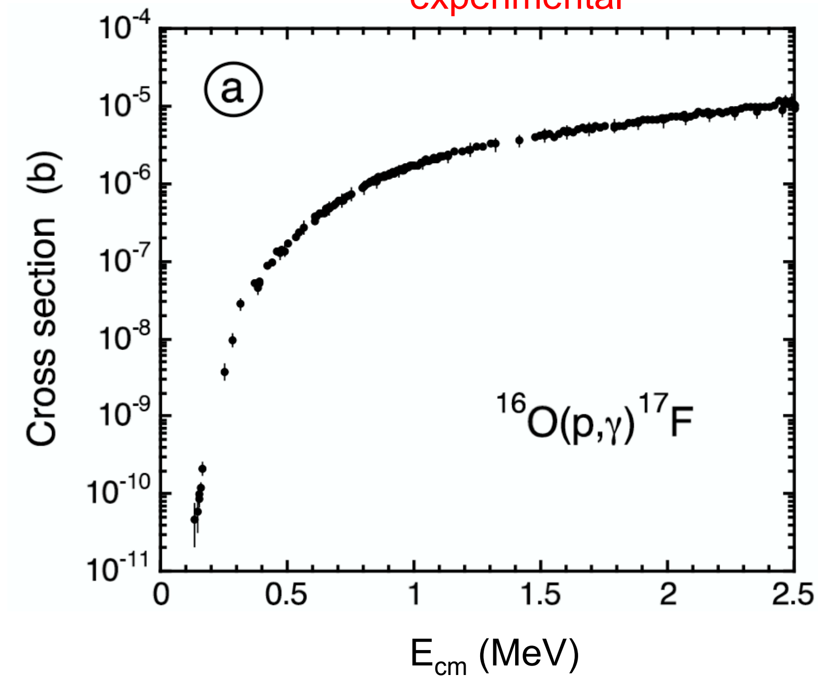
(after lengthy algebra, and for the limit of low E)

"Tunnel effect"

calculated



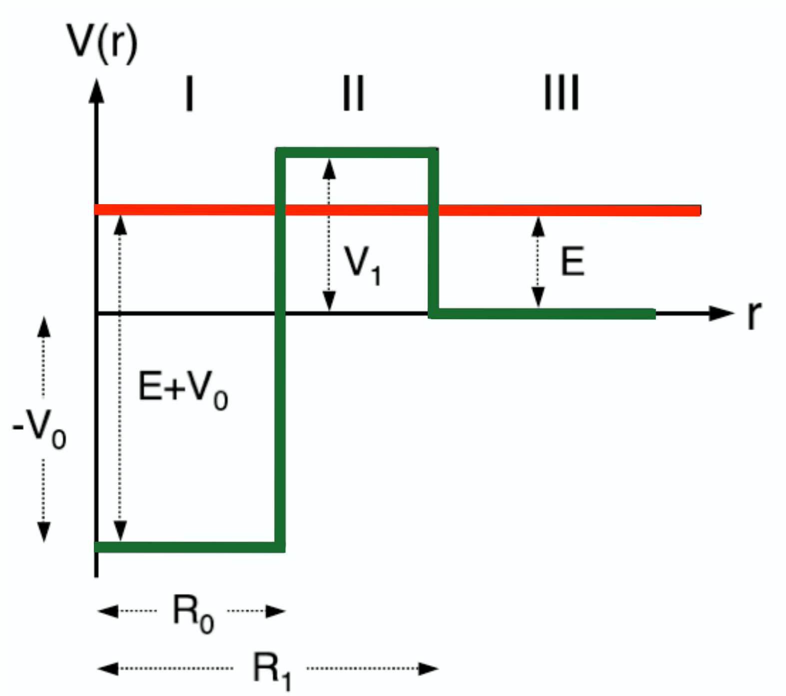
experimental



Tunnel effect is the reason for the strong drop in cross section at low energies!

Back to the simple potential, now in 3 dimensions

$$\lambda = \frac{2\pi}{K}$$



Wave function solutions:

$$u_I = A' \sin(Kr)$$
$$u_{II} = Ce^{-\kappa r} + De^{\kappa r}$$
$$u_{III} = F' \sin(kr + \delta_0)$$

$$K^2 = \frac{2m}{\hbar^2}(E + V_0)$$
$$\kappa^2 = \frac{2m}{\hbar^2}(V_1 - E)$$
$$k^2 = \frac{2m}{\hbar^2}E$$

Continuity condition…

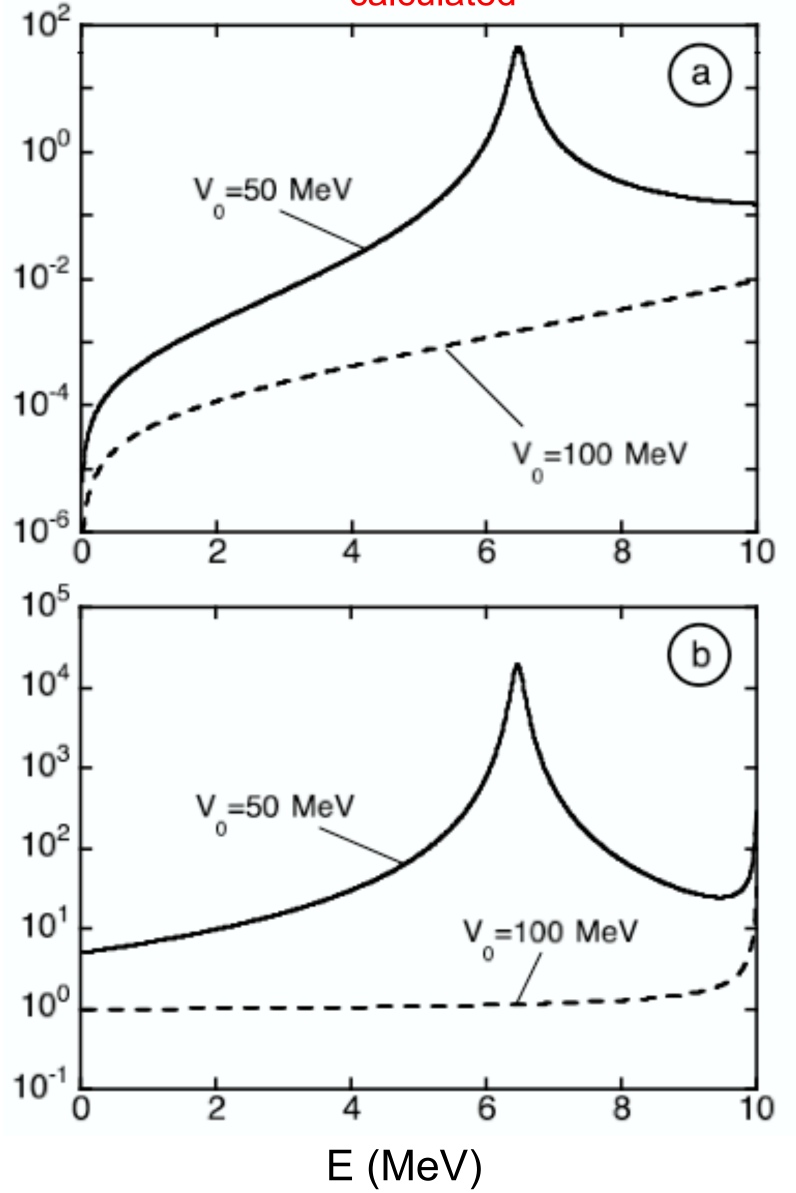
Wave intensity in interior region:
(after very tedious algebra)

$$\frac{|A'|^2}{|F'|^2} = \left\{ \sin^2(KR_0) + \left(\frac{K}{k}\right)^2 \cos^2(KR_0) + \sin^2(KR_0)\sinh^2(\kappa\Delta)\left[1 + \left(\frac{\kappa}{k}\right)^2\right] + \cos^2(KR_0)\sinh^2(\kappa\Delta)\left[\left(\frac{K}{\kappa}\right)^2 + \left(\frac{K}{k}\right)^2\right] \right.$$
$$\left. + \sin(KR_0)\cos(KR_0)\sinh(2\kappa\Delta)\left[\left(\frac{K}{\kappa}\right) + \left(\frac{K}{\kappa}\right)\left(\frac{\kappa}{k}\right)^2\right] \right\}^{-1}$$

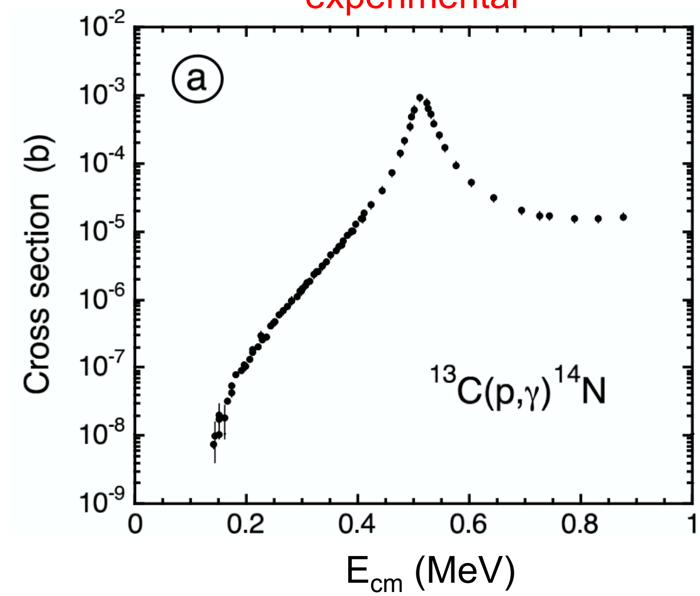
calculated

$$\frac{|A'|^2}{|F'|^2}$$

$$\frac{|A'|^2}{|F'|^2}\,\frac{1}{\hat{T}}$$



experimental

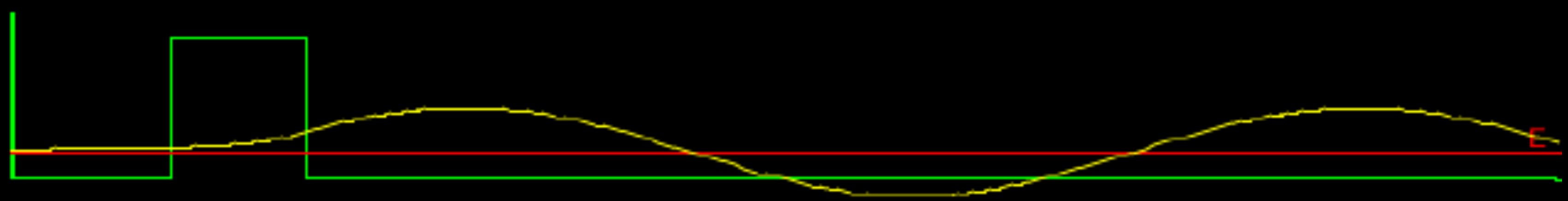


[change of potential depth V_0:
changes wavelength in interior region]

"Resonance phenomenon"

A resonance results from favorable wave function matching conditions at the boundaries!

Resonance phenomenon: radial wave function for varying potential depth V_0

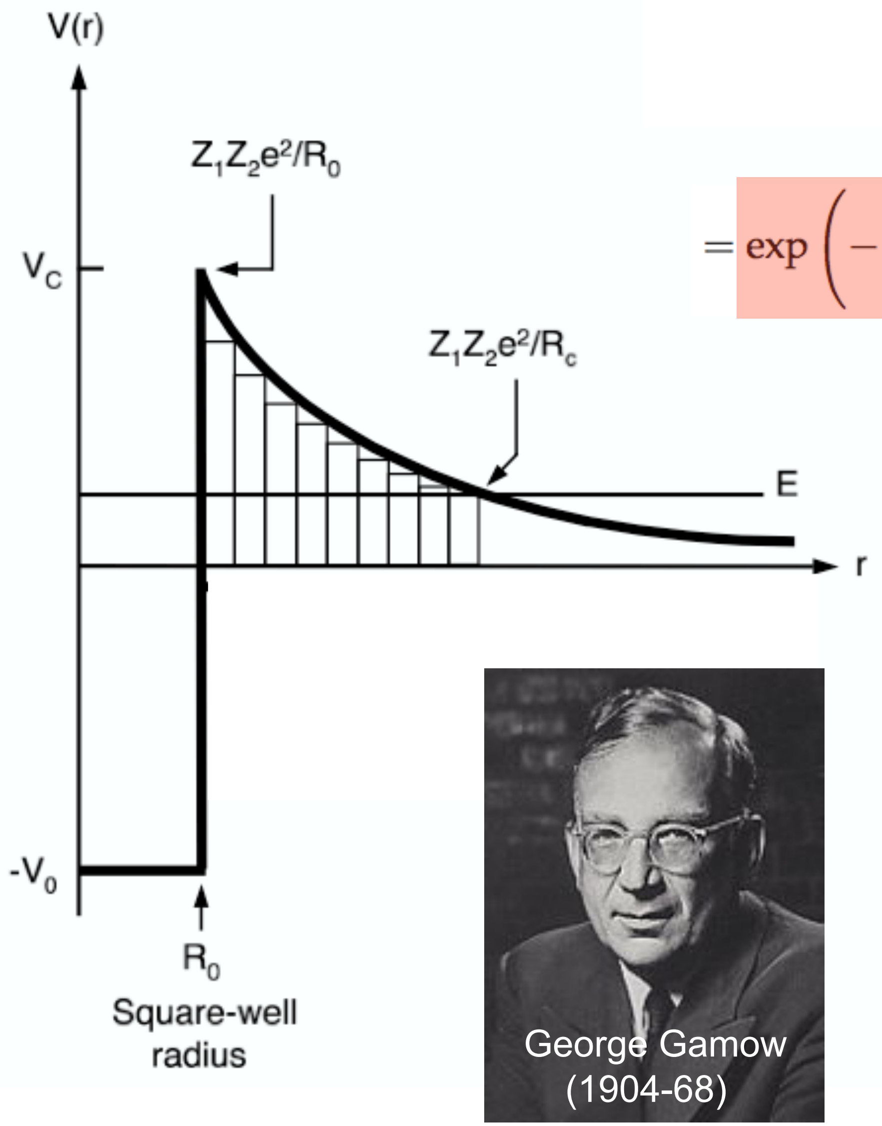


Transmission through the Coulomb barrier

$$\hat{T} = \hat{T}_1 \cdot \hat{T}_2 \cdot \ldots \cdot \hat{T}_n \approx \exp\left[-\frac{2}{\hbar}\sum_i \sqrt{2m(V_i - E)}(R_{i+1} - R_i)\right]$$

$$\xrightarrow[n\ \text{large}]{} \exp\left[-\frac{2}{\hbar}\int_{R_0}^{R_c} \sqrt{2m[V(r) - E]}\,dr\right]$$

$$= \exp\left(-\frac{2\pi}{\hbar}\sqrt{\frac{m}{2E}}Z_0 Z_1 e^2 \left[1 + \frac{2}{3\pi}\left(\frac{E}{V_C}\right)^{3/2}\right] + \frac{4}{\hbar}\sqrt{2mZ_0Z_1e^2R_0}\right)$$



[for low energies and zero angular momentum]

"Gamow factor" $e^{-2\pi\eta}$

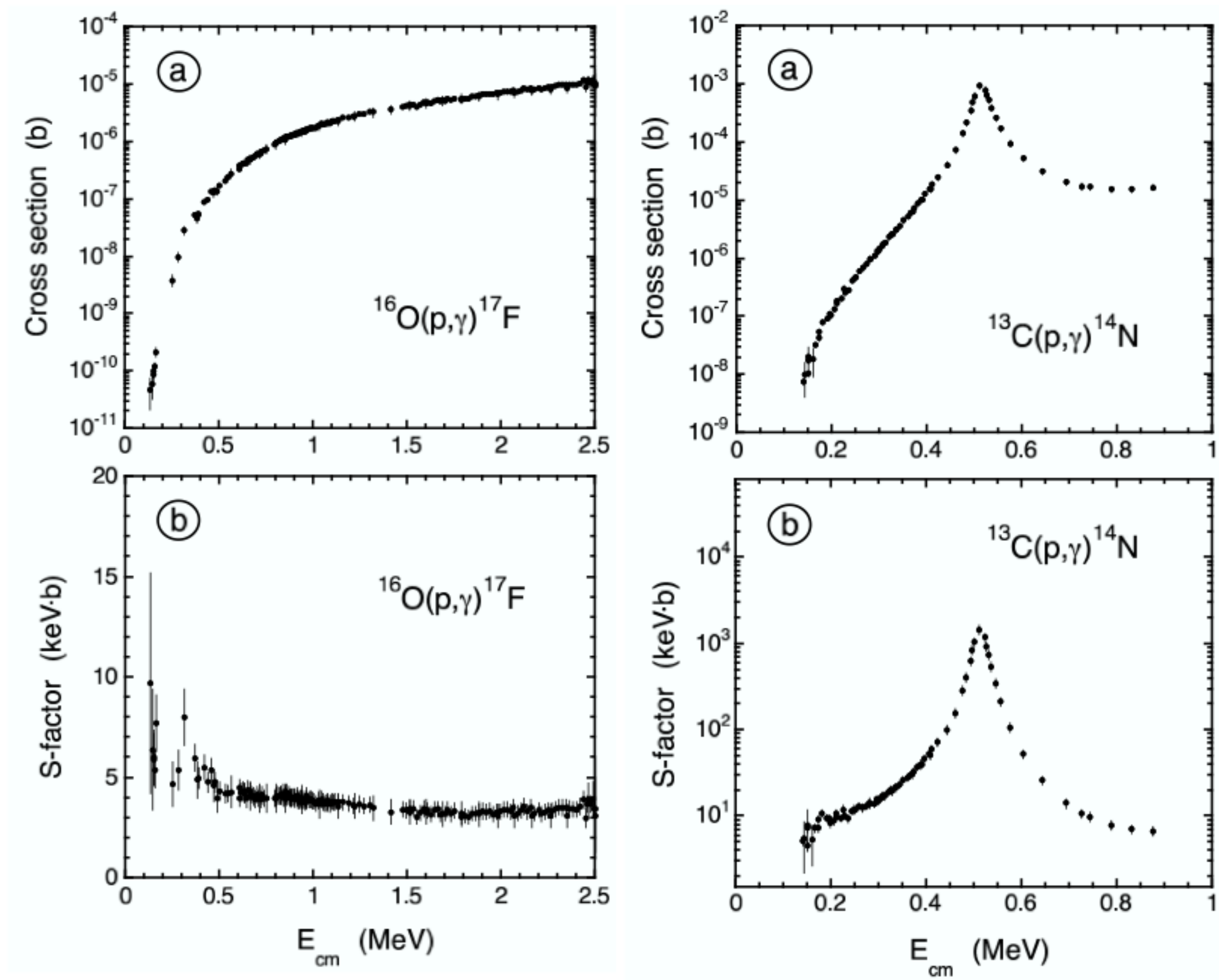
$$\sigma(E) \equiv \frac{1}{E}e^{-2\pi\eta}S(E)$$

"astrophysical S-factor"

Comparison: S-factors and cross sections

cross sections →

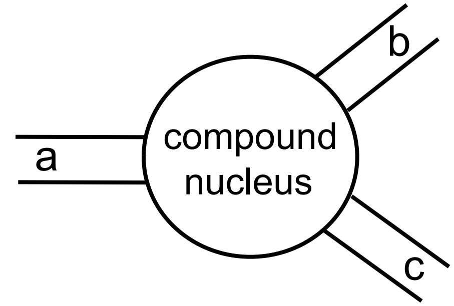
S-factors →



Formal reaction theory: Breit-Wigner formula

Eugene Wigner
(1902-95)
Nobel Prize 1963

Example: $^{22}Ne(\alpha,\gamma)^{26}Mg$
$^{22}Ne(\alpha,n)^{25}Mg$



$$\sigma_{\mathrm{BW}}(E) = \frac{\lambda^2}{4\pi} \frac{(2J+1)(1+\delta_{01})}{(2j_0+1)(2j_1+1)} \frac{\Gamma_a \Gamma_b}{(E_r - E)^2 + \Gamma^2/4}$$

de Broglie wavelength

partial widths for incoming and outgoing channel

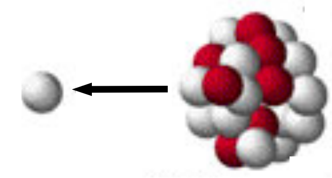
spin factor

resonance energy

total width

Used for:
- for fitting data to deduce resonance properties
- for “narrow-resonance” thermonuclear reaction rates
- for extrapolating cross sections when no measurements exist
- for experimental yields when resonance cannot be resolved

What are "partial widths"?



probability per unit time for formation or decay of a resonance (in energy units)

For protons/neutrons:

$$\Gamma_{\lambda c} = 2\gamma_{\lambda c}^2 P_c = 2\frac{\hbar^2}{mR^2} C^2 S\ \theta_{pc}^2\ P_c$$

A partial width can be factored into 3 probabilities:

C^2S: probability that nucleons will arrange themselves in a "residual nucleus + single particle" configuration ["spectroscopic factor"]

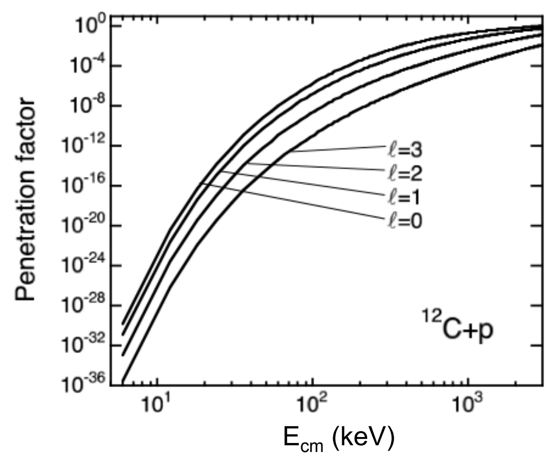
θ^2: probability that single nucleon will appear on nuclear boundary ["dimensionless reduced single particle width"; Iliadis, Nucl. Phys. A 618, 166 (1997)]

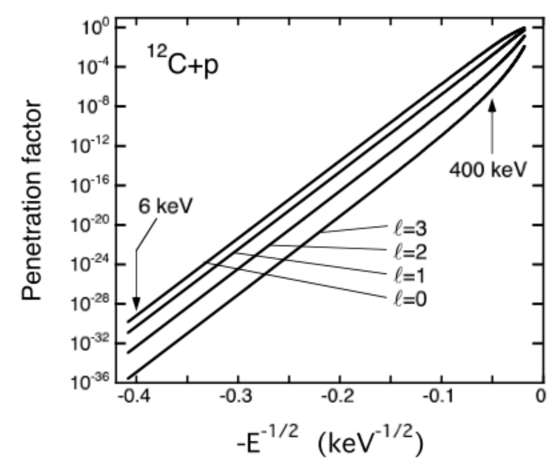
P_c: probability that single nucleon will penetrate Coulomb and centripetal barriers ["penetration factor"]

strongly energy-dependent:

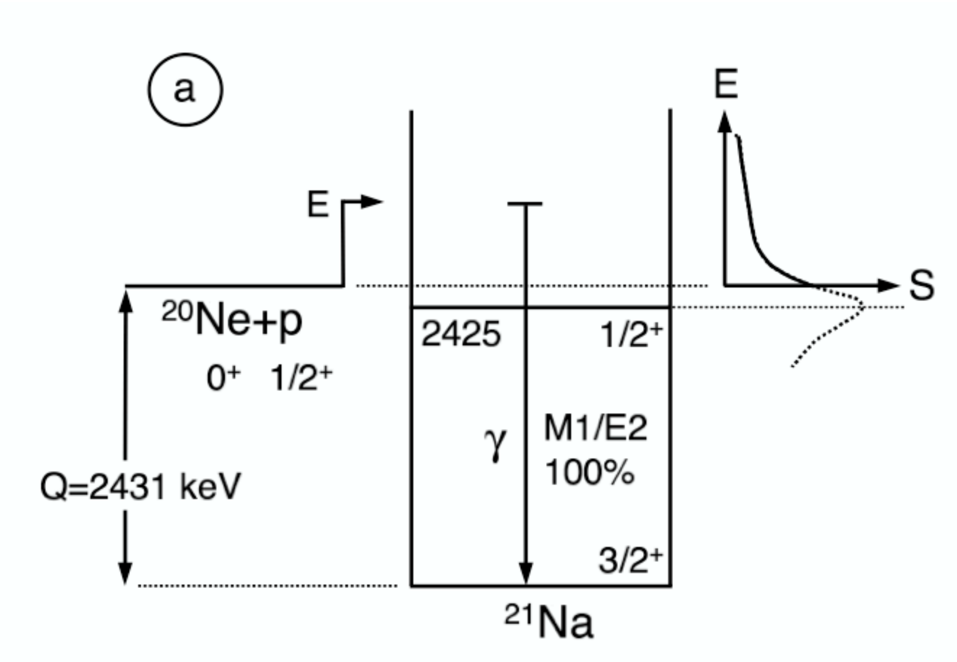
$$P_\ell = R\left(\frac{k}{F_\ell^2 + G_\ell^2}\right)_{r=R}$$

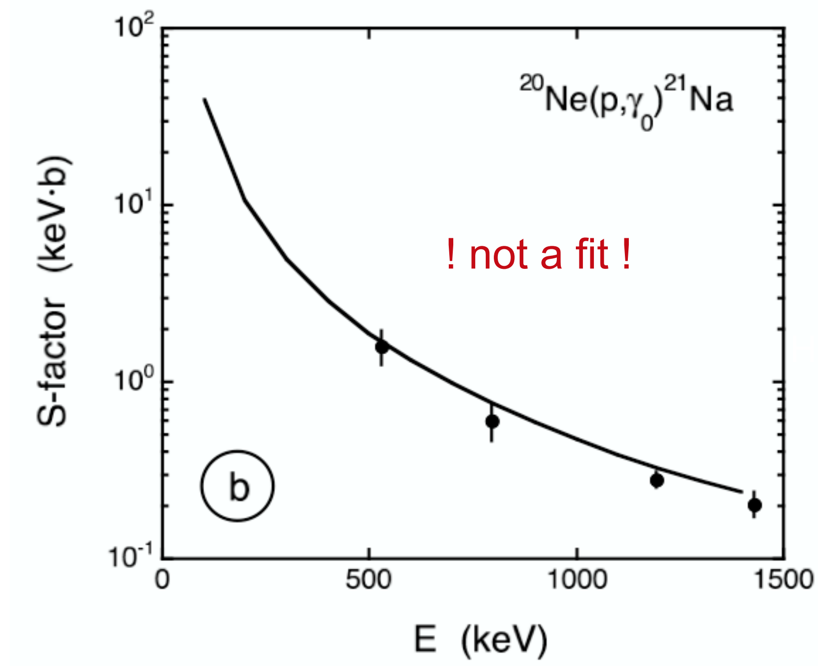
$$\propto e^{-2\pi\eta} = e^{-const/\sqrt{E}}$$





Sensitive test of the Breit-Wigner formula





[Data points from Rolfs & Rodney, Nucl. Phys. A 241, 460 (1975)]

- resonance energy obtained from known excitation energy
- proton partial width: estimated using C^2S from proton transfer
- γ-ray partial width estimated from measured lifetime (0.30 eV)

Breit-Wigner formula predicts accurately cross section extrapolated over 10^6 resonance widths!

Thermonuclear reactions

For a reaction 0 + 1 → 2 + 3 we find from the definition of σ (see earlier) a “reaction rate”:

$$r_{01} = N_0 N_1 \int_0^\infty v P(v) \sigma(v)\, dv \equiv N_0 N_1 \langle \sigma v \rangle_{01}$$

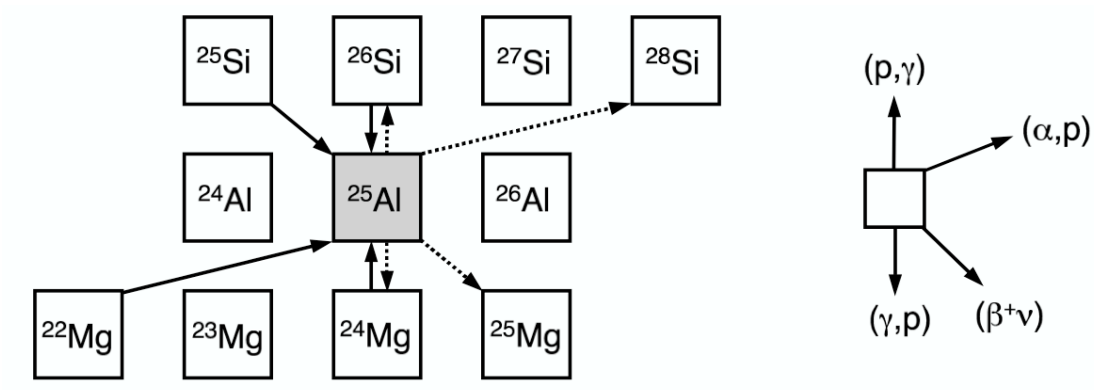
For a stellar plasma: kinetic energy for reaction derives from thermal motion:

“Thermonuclear reaction”

For a Maxwell-Boltzmann distribution:

$$\langle \sigma v \rangle_{01} = \left(\frac{8}{\pi m_{01}} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E\, \sigma(E)\, e^{-E/kT}\, dE$$

The interplay of many different nuclear reactions in a stellar plasma



$$
\begin{aligned}
\frac{d(N_{^{25}\mathrm{Al}})}{dt} =& N_{\mathrm{H}} N_{^{24}\mathrm{Mg}} \langle \sigma v \rangle_{^{24}\mathrm{Mg(p,\gamma)}} + N_{^{4}\mathrm{He}} N_{^{22}\mathrm{Mg}} \langle \sigma v \rangle_{^{22}\mathrm{Mg}(\alpha,\mathrm{p})} \\
&+ N_{^{25}\mathrm{Si}} \lambda_{^{25}\mathrm{Si}(\beta^{+}\nu)} + N_{^{26}\mathrm{Si}} \lambda_{^{26}\mathrm{Si}(\gamma,\mathrm{p})} + \ldots \\
&- N_{\mathrm{H}} N_{^{25}\mathrm{Al}} \langle \sigma v \rangle_{^{25}\mathrm{Al(p,\gamma)}} - N_{^{4}\mathrm{He}} N_{^{25}\mathrm{Al}} \langle \sigma v \rangle_{^{25}\mathrm{Al}(\alpha,\mathrm{p})} \\
&- N_{^{25}\mathrm{Al}} \lambda_{^{25}\mathrm{Al}(\beta^{+}\nu)} - N_{^{25}\mathrm{Al}} \lambda_{^{25}\mathrm{Al}(\gamma,\mathrm{p})} - \ldots
\end{aligned}
$$

} production (first two lines)

} destruction (last two lines)

System of coupled differential equations: "nuclear reaction network"

Solved numerically
[Arnett, "Supernovae and Nucleosynthesis", Princeton University Press, 1996]

Special case #1: reaction rates for smoothly varying S-factors ("non-resonant")

$$\sigma(E) \equiv \frac{1}{E} e^{-2\pi\eta} S(E)$$

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty E\,\sigma(E)\, e^{-E/kT}\, dE$$

$$= \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} S_0 \int_0^\infty e^{-2\pi\eta}\, e^{-E/kT}\, dE$$

"Gamow peak"

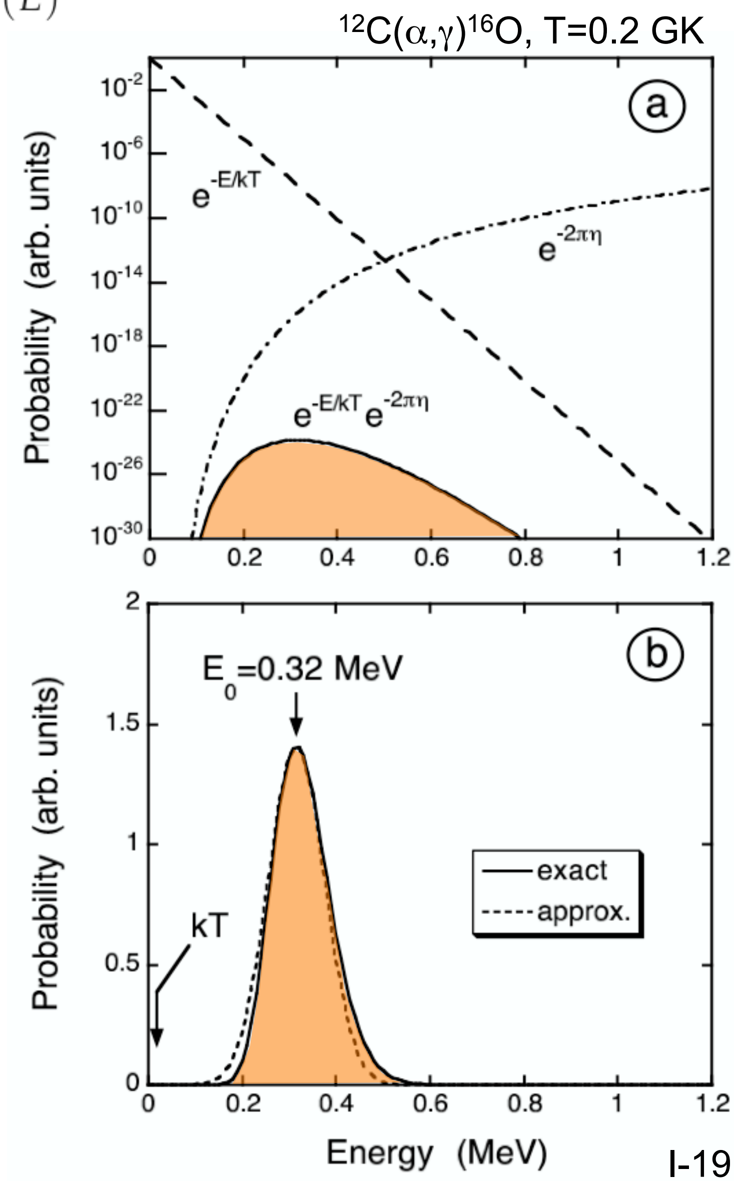
Represents the energy range over which most nuclear reactions occur in a plasma!

Location and 1/e width of Gamow peak:

$$E_0 = \left[\left(\frac{\pi}{\hbar}\right)^2 \left(Z_0 Z_1 e^2\right)^2 \left(\frac{m_{01}}{2}\right) (kT)^2\right]^{1/3}$$

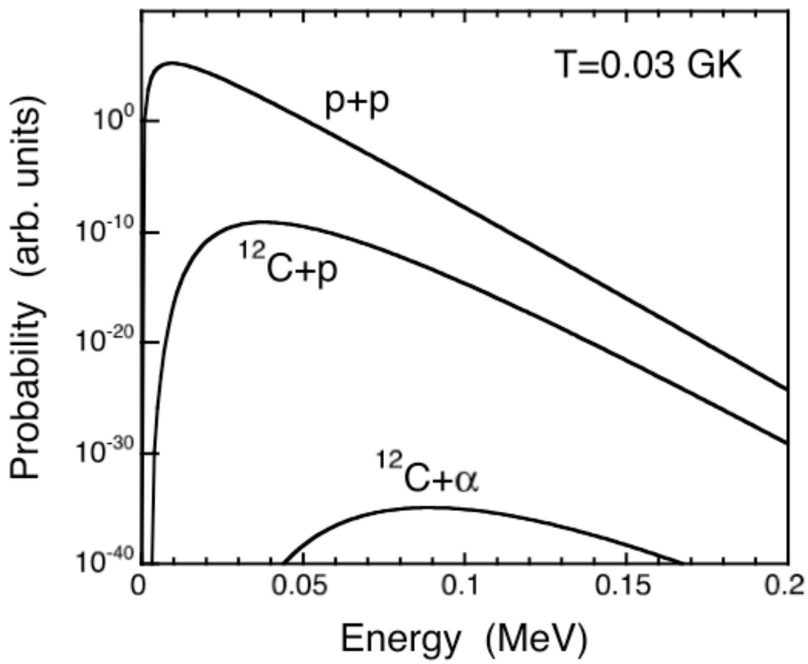
$$= 0.1220 \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0 + M_1} T_9^2\right)^{1/3} \quad \text{(MeV)}$$

$$\Delta = \frac{4}{\sqrt{3}}\sqrt{E_0 kT} = 0.2368 \left(Z_0^2 Z_1^2 \frac{M_0 M_1}{M_0 + M_1} T_9^5\right)^{1/6} \quad \text{(MeV)}$$



however, see: Newton, Iliadis et al., Phys. Rev. C 045801 (2007)

Gamow peaks

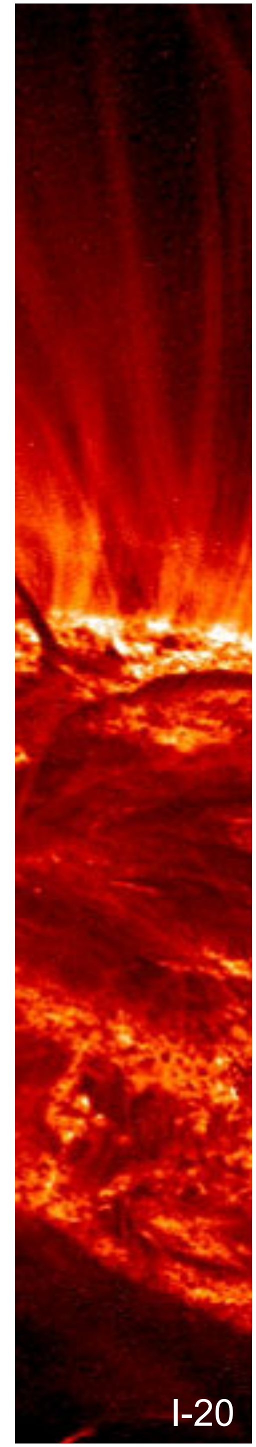


Important aspects:

(i) Gamow peak shifts to higher energy for increasing charges Z_p and Z_t

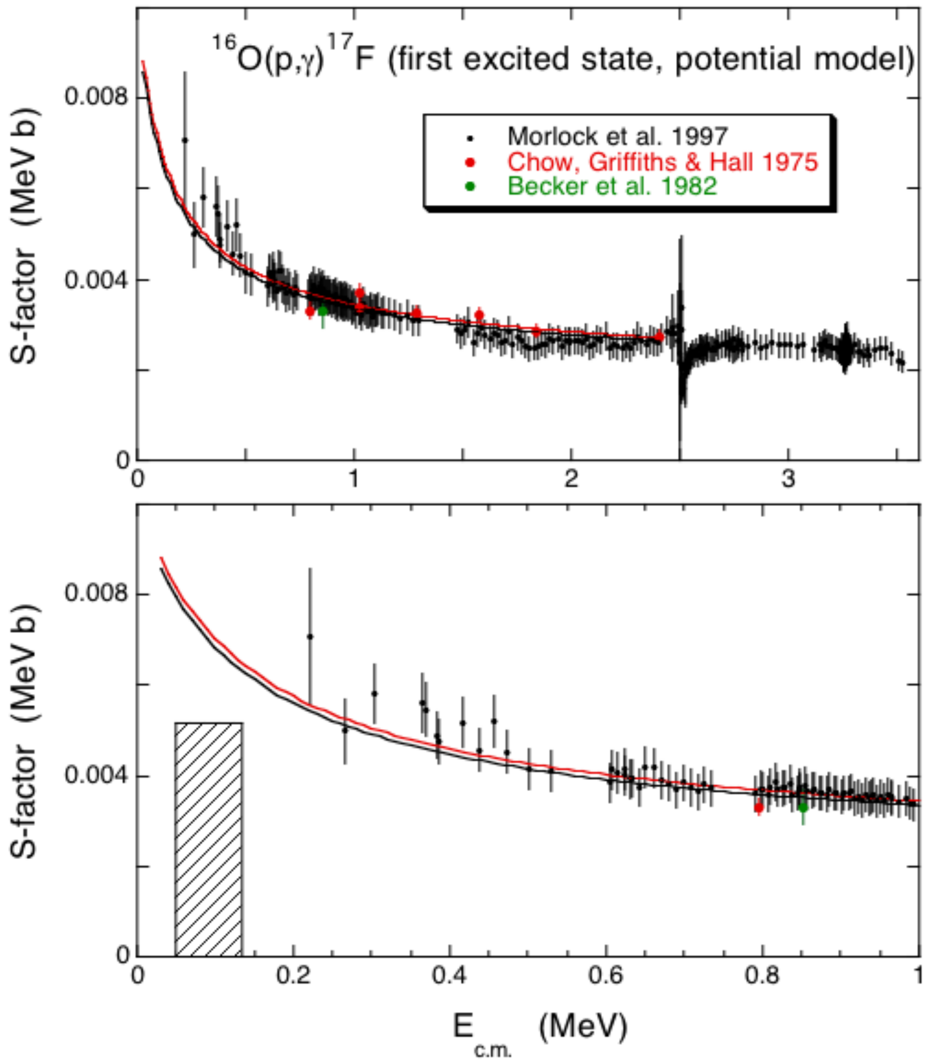
(ii) at same time, area under Gamow peak decreases drastically

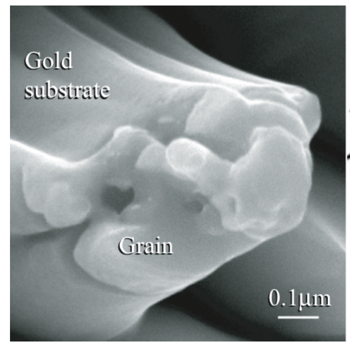
Conclusion: for a mixture of different nuclei in a plasma, those reactions with the smallest Coulomb barrier produce most of the energy and are consumed most rapidly [→ stellar burning stages, see Lectures #3 and #4]

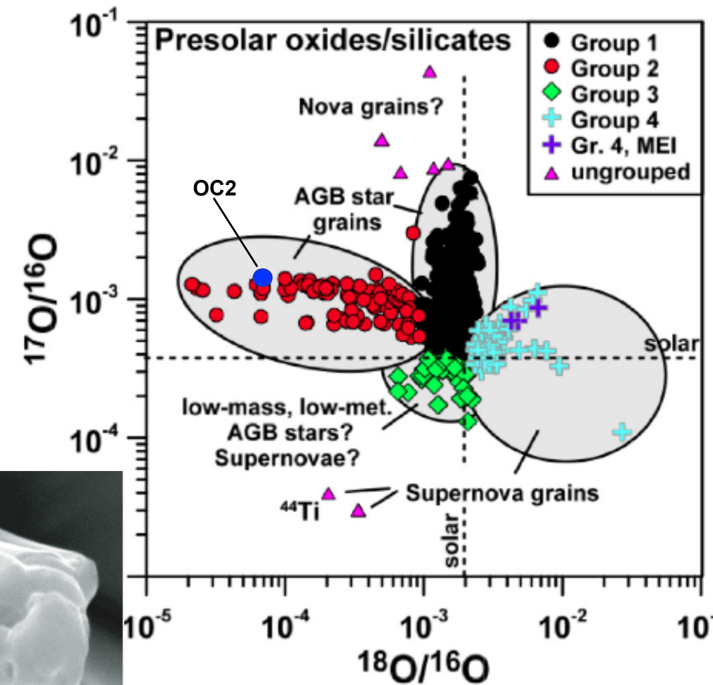


Example for smoothly varying σ: $^{16}O(p,\gamma)^{17}F$ direct capture

Iliadis, Angulo, Descouvemont, Lugaro and Mohr, Phys. Rev. C 77, 045802 (2008)







Reaction important for $^{17}O/^{16}O$ ratio predicted by models of massive AGB stars

New accurate cross section shows: there is presently no clear evidence of a **massive** AGB star origin for any observed stellar grain

Precise reaction rates are crucial!

Special case #2: reaction rates for "narrow" resonances (Γ_i const over total Γ)

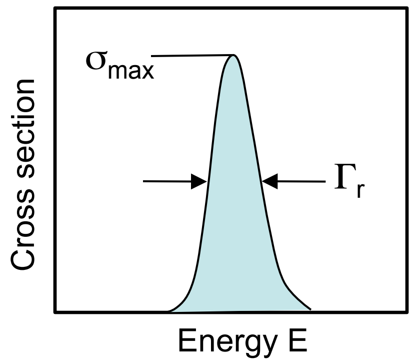
Breit-Wigner formula (energy-**independent** partial widths)

$$N_A \langle \sigma v \rangle = \left(\frac{8}{\pi m_{01}} \right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty E\, \sigma(E)\, e^{-E/kT}\, dE$$

$$= N_A \frac{\sqrt{2\pi}\hbar^2}{(m_{01} kT)^{3/2}} e^{-E_r/kT} \omega \frac{\Gamma_a \Gamma_b}{\Gamma} 2\pi$$

resonance energy needs to be known rather precisely [takes into account only rate contribution at E_r]

"resonance strength" $\omega\gamma$: proportional to area under narrow resonance curve



$$\omega\gamma \propto \sigma_{max} \cdot \Gamma_r$$

Special case #3: reaction rates for "broad resonances"

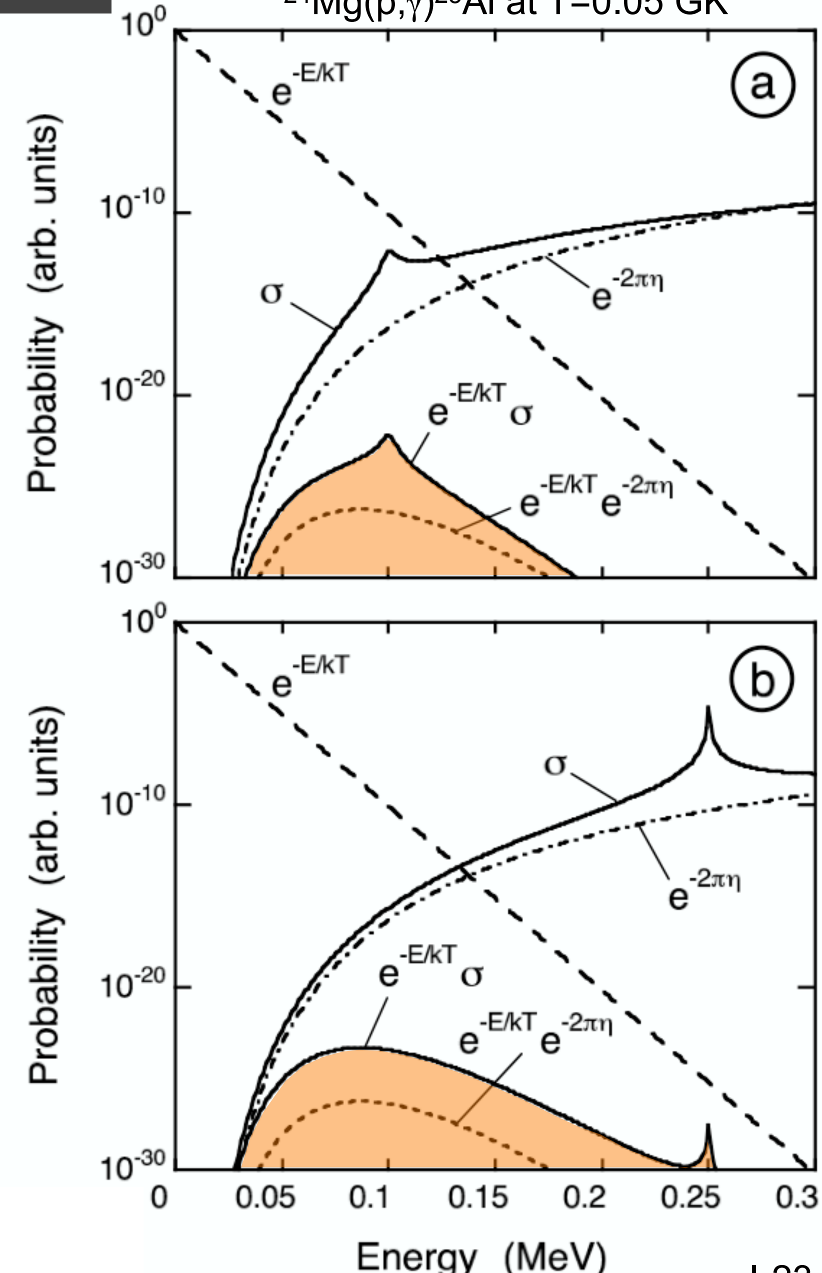
Breit-Wigner formula (energy-**dependent** partial widths)

$$N_A\langle\sigma v\rangle = \left(\frac{8}{\pi m_{01}}\right)^{1/2} \frac{N_A}{(kT)^{3/2}} \int_0^\infty E\,\sigma(E)\,e^{-E/kT}\,dE$$

rate can be found from numerical integration

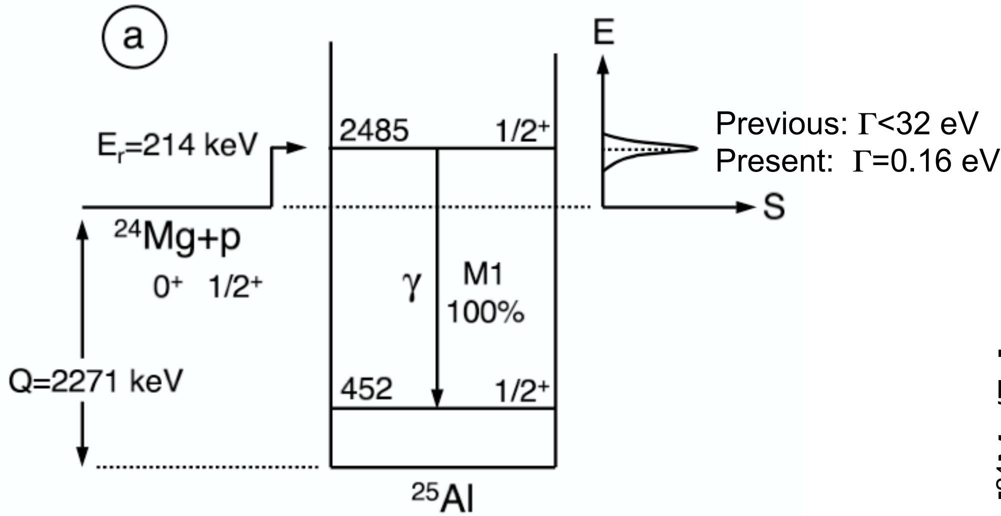
There are two contributions to the rate:
(i) from "narrow resonance" at E_r
(ii) from tail of broad resonance

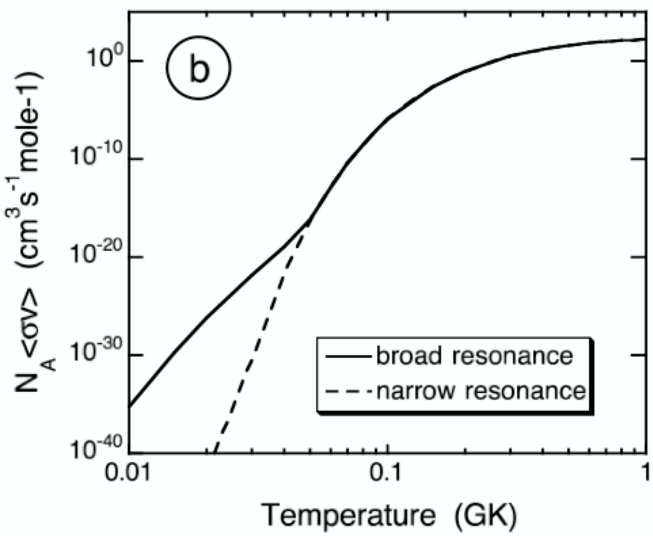
$^{24}Mg(p,\gamma)^{25}Al$ at T=0.05 GK



Example for resonance: E_r=214 keV in $^{24}Mg(p,\gamma)^{25}Al$

Powell, Iliadis et al., Nucl. Phys. A 660, 349 (1999)





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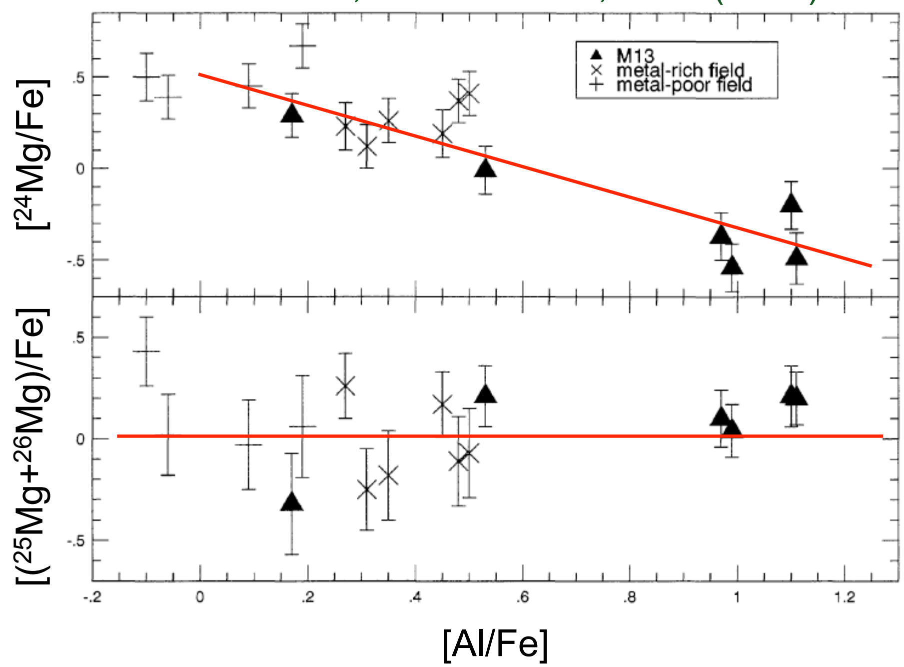
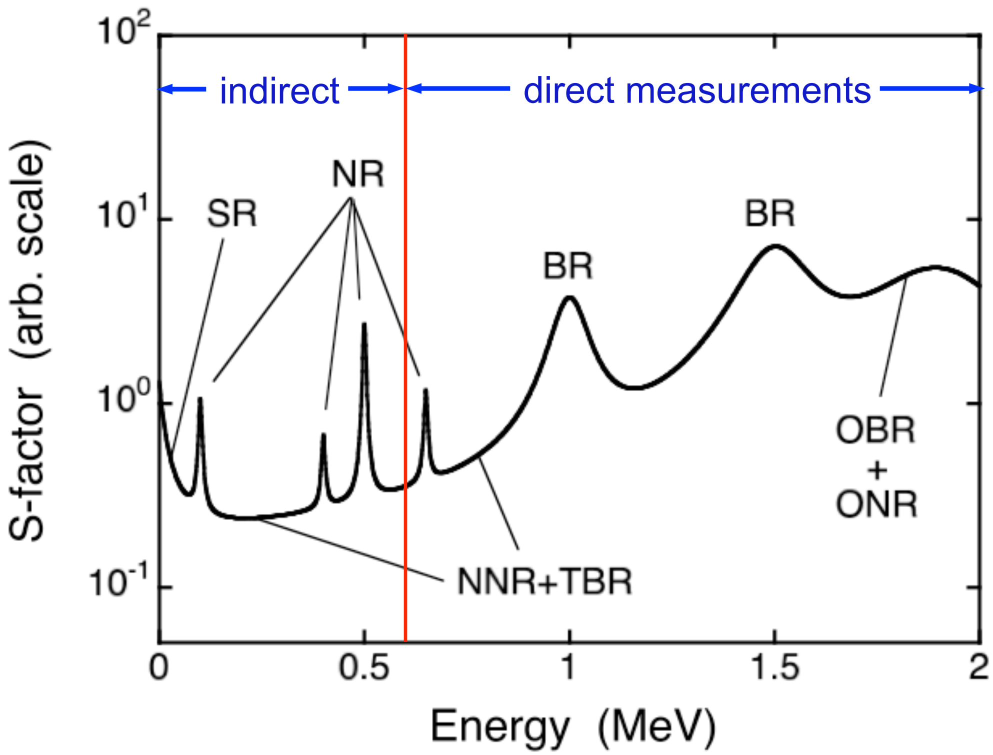


Conclusion: ^{24}Mg-Al anticorrelation cannot be produced in globular cluster red giants at 50 MK

Total thermonuclear reaction rate



Need to consider:

- non-resonant processes
- narrow resonances
- broad resonances
- subthreshold resonances
- interferences
- continuum

every nuclear reaction represents a special case !