Indirect Methods in Nuclear Astrophysics

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Indirect methods in Nuclear Astrophysics

Introduction

Nuclear Astrophysics ⇒ most important subfield of applied Nuclear Physics

A truly interdisplinary field which combines astronomical observations and the Astrophysical modeling with the Nuclear Physics measurements and theory

It concentrates on understanding the primordial and stellar nucleosynthesis, stellar evolution and interpretation of the cataclysmic stellar events like novae and supernovae.

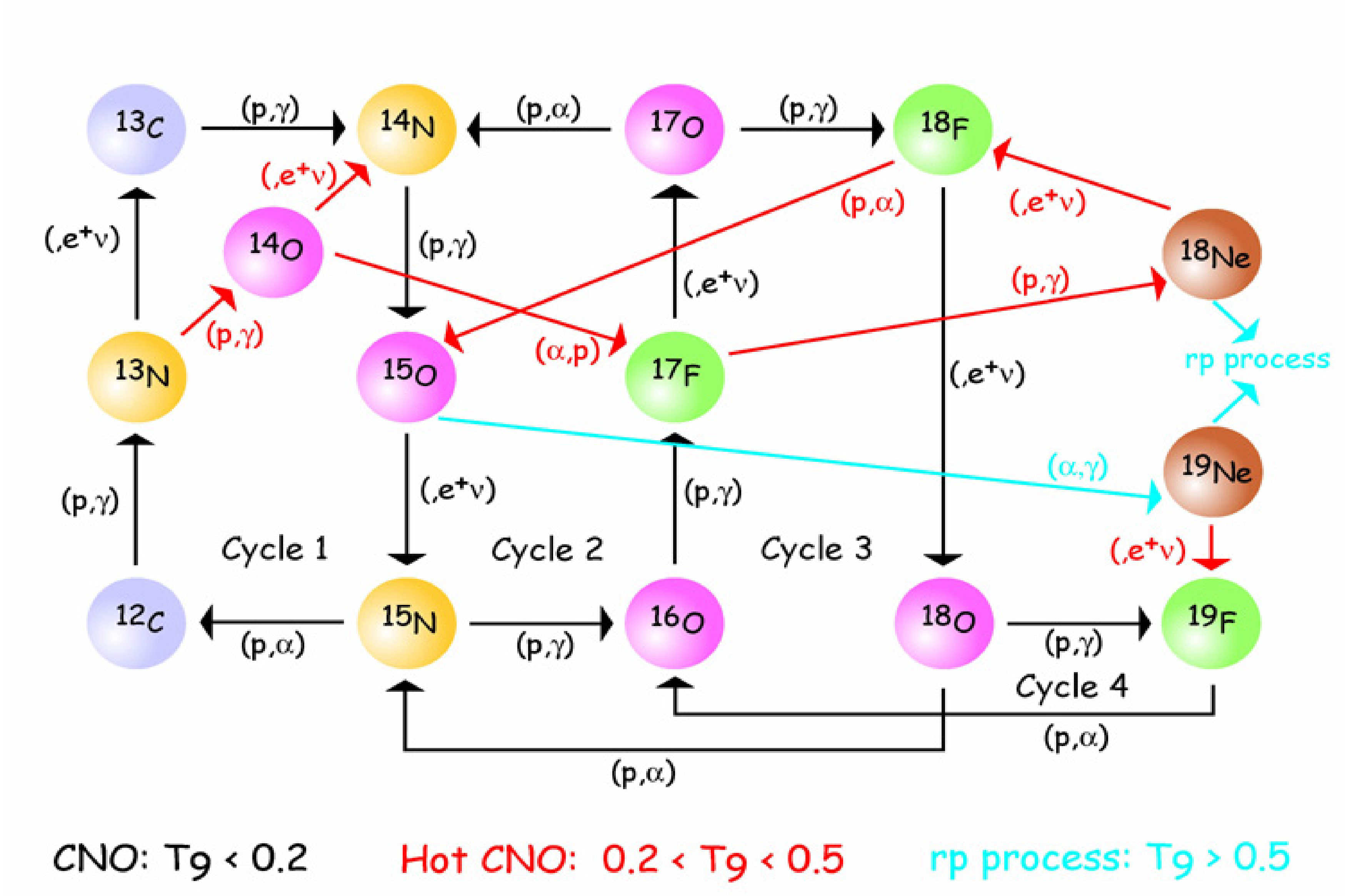
Units of Energy scales in Nuclear Physics and Astrophysics

In Astrophysics usually the units are 10^6 K = 0.0862 keV

Temperatures in Solar interior is 15.10^6 K which is 1.3 keV.

Stars Burn their nuclear fuel at very low energies ⇒ cross sections are small

Thermonuclear reactions play a key role in our quest for understanding the astrophysical phenomena



Another example where nuclear reactions play a key role

THE pp CHAIN in the SUN

$p + p \longrightarrow d + e^+ + \nu_e$ OR $p + e^- + p \longrightarrow d + \nu_e$

$d + p \longrightarrow {}^3He + \gamma$

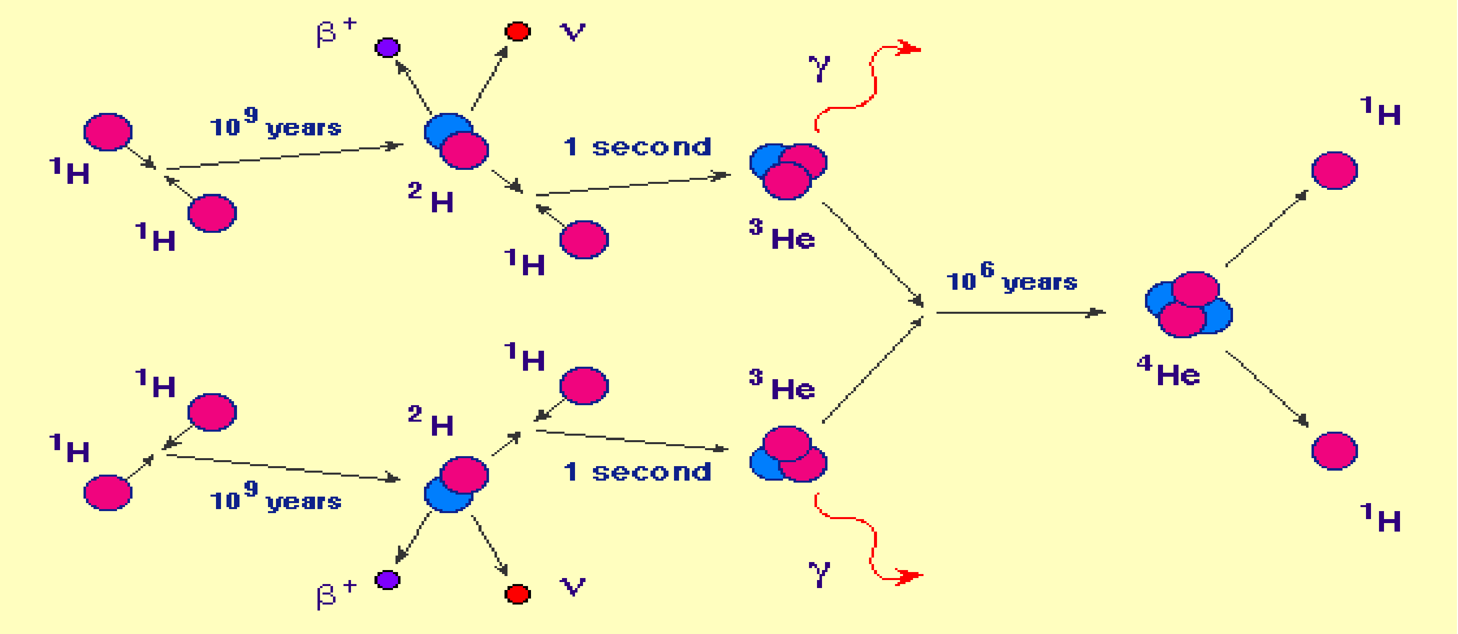
${}^3He + {}^3He \longrightarrow \alpha + 2p$ OR ${}^3He + \alpha \longrightarrow {}^7Be + \gamma$

${}^7Be + e^- \longrightarrow {}^7Li + \nu_e$

${}^7Li + p \longrightarrow 2\alpha$ OR ${}^7Be + p \longrightarrow {}^8B + \gamma$

${}^8B \longrightarrow {}^8Be^* + e^+ + \nu_e$

${}^8Be^* \longrightarrow 2\alpha$



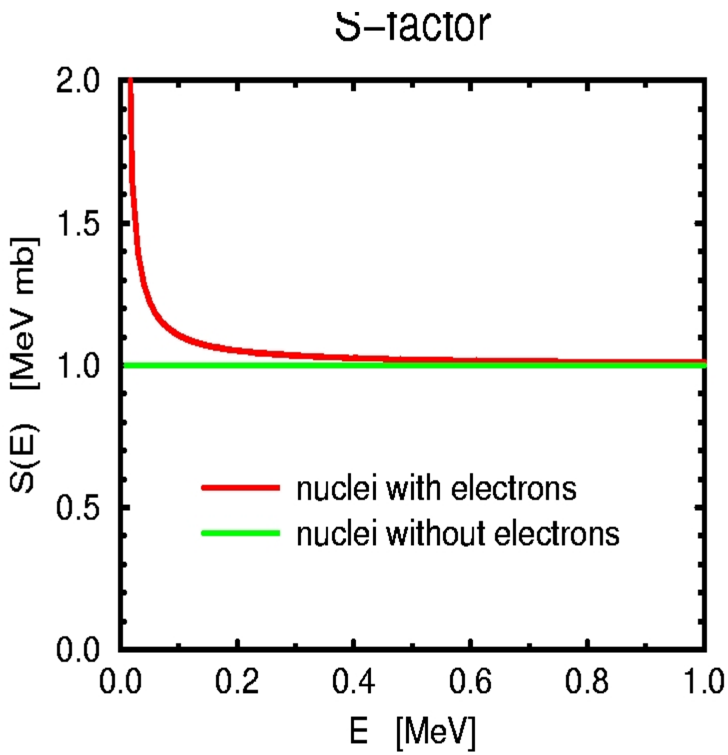
Generally

Nuclear Astrophysics involves long measurements of small cross sections at lower and lower energies

- **Accelerators capable of producing very high intensity beams**
- **Efficient back ground suppression**
- **Very high resolutions**
- **Electron Screening Corrections**

Lab. cross sections are measured on targets which are in the form of atoms.

In astrophysical models cross sections on bare nuclei are required.



Extrapolation of the measured data to actual astrophysical energies has considerable uncertainties

Energy Scales and Reaction Rates

Reaction rate of a reaction in stellar gas is defined as $N_x N_y \underline{v\sigma(v)}$. → Fold this with a velocity distribution probability function

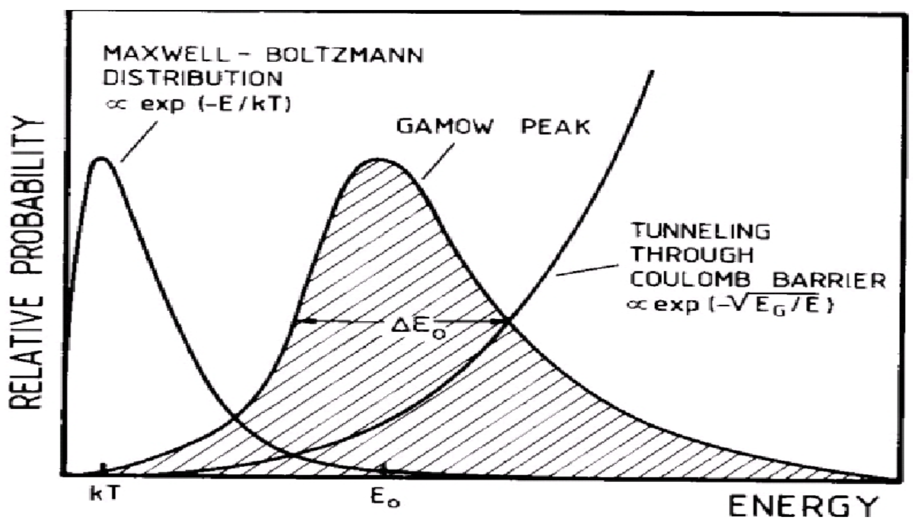
$$<\sigma v> = (8/\pi\mu)^{1/2} (1/kT)^{3/2} \int \sigma(E) E \exp(-E/kT) dE$$

Maxwell-Boltzmann distribution

Astrophysical S-factor: $S(E) = \sigma(E) E \exp(2\pi\eta)$, $\eta = Z_1 Z_2 e^2/\hbar v$

Charged particle induced Non-resonant reaction

$$<\sigma v> = (8/\pi\mu)^{1/2} (1/kT)^{3/2} \int S(E) E \exp(-E/kT - b/E^{1/2}) dE$$



$b = 0.989\, Z_1 Z_2\, \mu^{1/2} (\mathrm{MeV})^{1/2}$

$E_0 = 1.22\, (Z_1 Z_2\, \mu^{1/2})^{2/3}\, T_6^{2/3}$ keV

$p + p$	5.9 keV	$T_6 = 15$
$p + {}^{14}N$	26.5 keV	
$\alpha + {}^{12}C$	56.0 keV	

Reactions through isolated resonances

$$<\sigma v> = (2/\pi\mu T)^{1/2} \exp(-E/kT) \eta^2 \omega\, \Gamma_a \Gamma_b / \Gamma,$$

Resonance parameters

High energy reactions are commonly applied to determine the resonance widths and energies.

INDIRECT METHODS

The major problem in nuclear astrophysics: E_0 is generally at energies too low for direct measurements of the cross sections.

Dependence on extrapolation formula or procedure

Recent developments often allow to obtain equivalent information from experiments performed with higher energy beams. Extrapolation is made redundant.

High energy experiments yield higher events rates. Experimental conditions are relatively less stringent.

These methods necessarily depend on theoretical inputs having uncertainties of their own. So two sources of errors.

The overall uncertainty can be reduced by combining various approaches

Some Indirect approaches

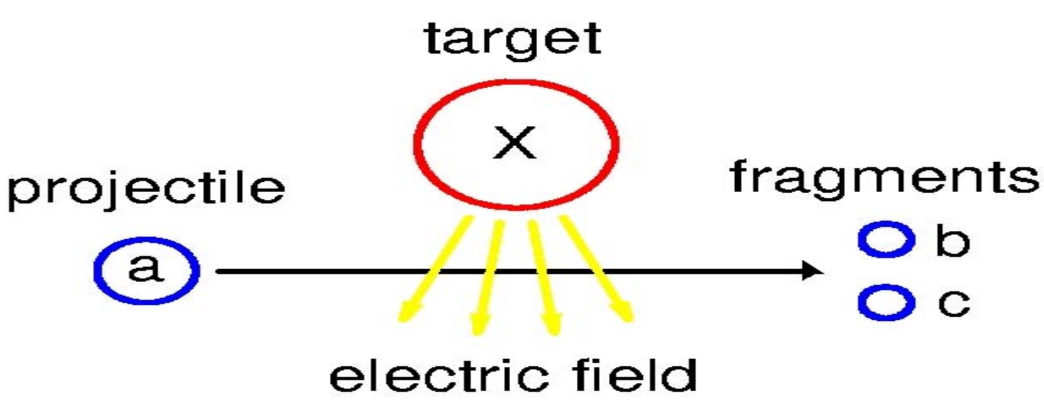
Coulomb Dissociation Method (Baur, Bertulani, Rebel, 1986)

Radiative capture reaction, $b + c \rightarrow a + \gamma$ can be studied by the time reversed reaction, $\gamma + a \rightarrow b + c$. The two reactions are related by

$$\sigma(b + c \rightarrow a + \gamma) = [2(2j_a + 1)/(2j_b + 1)(2j_c + 1)]\,(k_\gamma^2/k_{CM}^2)\,\sigma(a + \gamma \rightarrow b + c)$$

$k_\gamma/k_{CM} \ll 1$, phase space favors a strong enhancement of the reverse reaction

- $\gamma + a \rightarrow b + c$ reaction with a real photon beam, Utsunomia et al, PRC 63 (2001)
- **Use the equivalent photon spectrum, provided by the Coulomb field of a target nucleus in the fast peripheral collision.**



Projectile is excited to the continuum which decays into fragments b & c.

COULOMB DISSOCIATION METHOD

ADVANTAGES

1. Breakup reaction can be performed at higher energies (larger cross sec).

2. At higher projectile energies fragments emerge with larger velocities.(detection and target thickness conditions are less stringent).

3. Adequate kinematical conditions of coincidence (b-c) measurements allow to study extremely low (E_{bc}) with high precision.

BUT SATISFY FOLLOWING CONDITIONS

1. Influence of the strong nuclear field on the projectile motion. (Ensure that it is negligible).

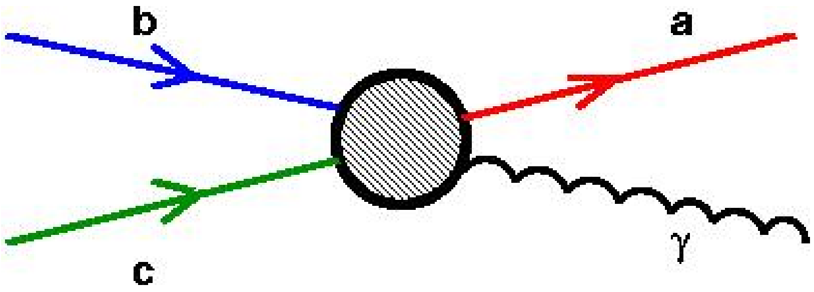
2. Projectile excitation (or breakup) is dominated by single multipolarity.

3. Theory related issues $\Rightarrow$ accuracy of the first order theory, have a control over the higher order effects (see that they are negligible)

ASYMPTOTIC NORMALIZATION CONSTANT (ANC) METHOD

H.M. Xu et al., PRL 73 (1994) 2027

Direct capture reaction $b + c \rightarrow a + \gamma$



$\sigma \propto |M|^2$

$\propto$ |Overlap of the bound state wave functions|^2

If this reaction is peripheral,

$\propto$ |C(ANC) X Whittaker function|^2

C(ANC) is sufficient to calculate the capture cross section and hence the S-factor.

How to determine C(ANC) ⇒ find a reaction where the same vertex enters, transfer reactions A (a,b) B(=A+c)

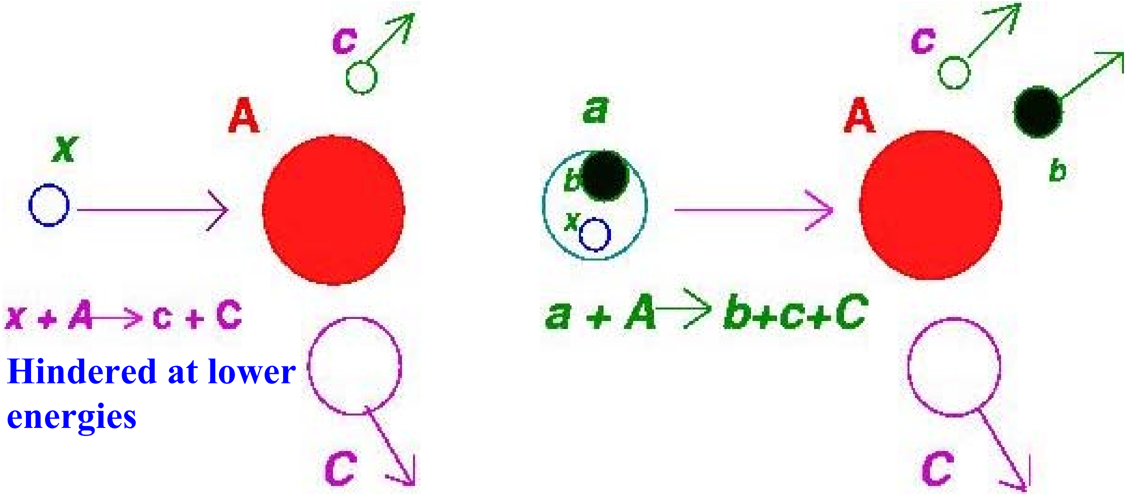
Choose kinematical conditions to ensure the peripheral condition

Calculate transfer cross sections as accurately as possible, uncertainties due to optical potentials, second bound state

C. TROJAN-HORSE METHOD

G. Baur, F. Roesel, D. Trautmann, R. Shyam, Phys. Rep. 111 (1984) 333

For any charged particle induced reaction at astrophysical energies



Hindered at lower energies

This reaction can proceed as *a* can have larger energy. *x* is brought in the reaction zone of target *A* hidden inside the projectile .

Highlights

- ***A* – *a* energy can be well above the Coulomb barrier**
- **Cross sections are large**
- **Low *A* - *x* energies are accessible, $E_a = E_b + E_x - Q$**

Ensure

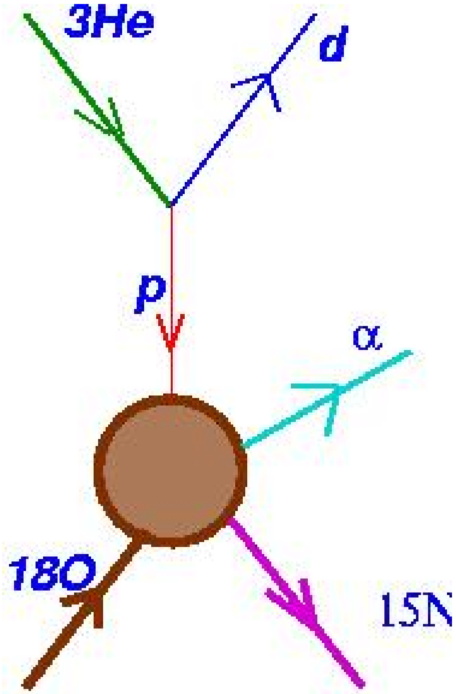
Accuracy of the calculations of the breakup reaction a + A → b + c + C

The Trojan - Horse Method

Example

$p + {}^{18}O \rightarrow \alpha + {}^{15}N$ CNO cycle Gammow peak ; 30 keV

We study this reaction by $^{3}He + {}^{18}O \rightarrow d + \alpha + {}^{15}N$



The information about the cross section

$$\sigma\,(xA \rightarrow cC) = (\pi/q_x^{\,2})\,\Sigma\,(2\lambda+1)\,|S_{\lambda c}|^2$$

can be obtained from the experimentally determined coincidence cross section $\mathbf{d^3\sigma / d\Omega_c\, d\Omega_b\, dE_b}$.

$$d^3\sigma / d\Omega_c\, d\Omega_b\, dE_b = (\text{phase space})\, |\Sigma_{\lambda m}\, T_{\lambda m} S_{\lambda c} Y_{\lambda m}\, (q_c)|^2$$

DWBA THEORY OF INELASTIC BREAKUP REACTIONS

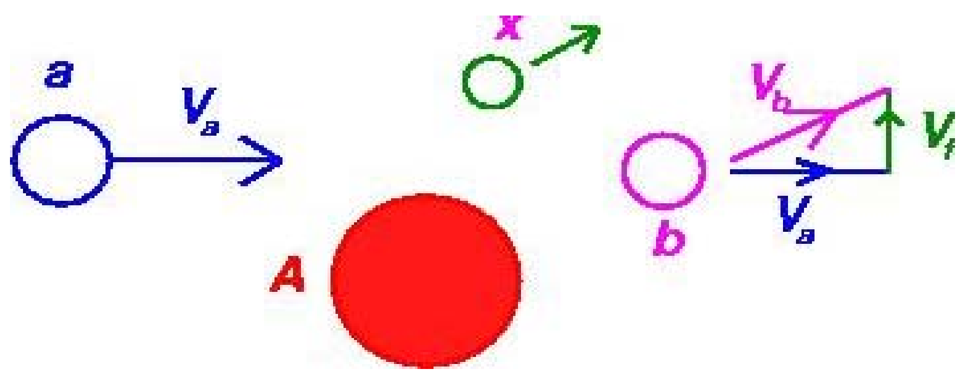
A Reminder of the Relevant Nuclear Reaction Theory

Breakup reactions: $a + A \rightarrow b + c + A$ **at least a 3-body problem**

Two modes of breakup

- **Elastic breakup (A remains in the g.s.)**
- **Inelastic breakup (c + A can go to c + C)**

Mechanism of breakup reactions (Elastic Breakup)

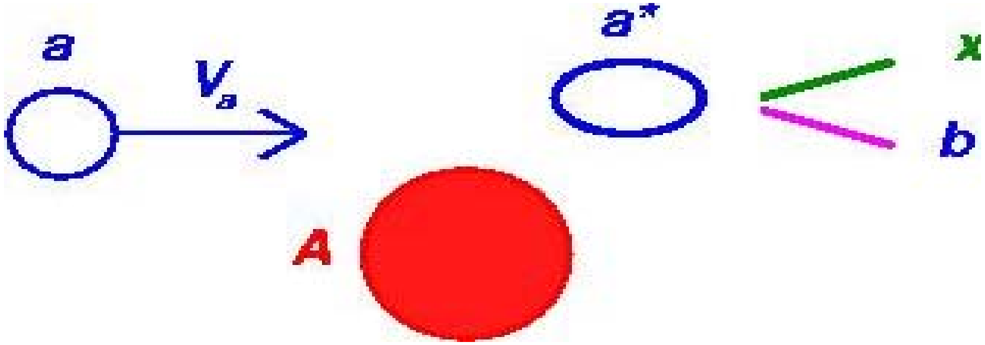


Spectator participant picture

$$T_{fi}^{(+)}(DWBA) = <\chi^{(-)}(q_c,r_{cA})\chi^{(-)}(q_b,r_{bA})|V_{bc}|\chi^{(+)}(q_a,r_{aA})\phi_a(r_b c)>$$

$$T_{fi}^{(-)}(DWBA) = <\chi^{(-)}(q_c,r_{cA})\chi^{(-)}(q_b,r_{bA})|V_{cA}+V_{bA}-U_{aA}|\chi^{(+)}(q_a,r_{aA})\phi_a(r_b c)>$$

Mechanism of Breakup reactions



Sequential Breakup picture

Different approximation for the Final state

$$T_{fi}^{(-)A}(\text{DWBA}) = \langle \chi^{(-)}(Q_f, r_{aA})\phi^{(-)}(q_f, r_{bc})|V_{cA}+V_b|\chi^{(+)}(q_a, r_{aA})\phi_a(r_b c)\rangle$$

$\phi^{(-)}(q_f, r_{bc}) \Rightarrow$ relative motion between b-c, q_f = relative momentum of b-c

Numerical computations are relatively simpler

More suitable when outgoing fragments are detected with very small relative Energies (in astrophysically interesting radiative fusion reactions).

When nuclear interactions are absent ⇒ Coulomb excitation of the projectile *a*

⇓

Semi-classical counterpart ⇒Alder-Winther theory

CDCC method ⇒ relative and cm motion of the fragments are not independent

Theory of Coulomb dissociation as applied to radiative fusion

Radiative capture cross sections are related to photo-disintegration

$$\sigma(a + \gamma \rightarrow b + c) = [(2j_b + 1)(2j_c + 1) / 2(2j_a + 1)]\ (k_{CM}{}^2 / k_\gamma{}^2)\ S(E)\ E\ \exp(-2\pi\eta)$$

Cross section for Coulomb excitation

$$d^2\sigma/d\Omega\, dE\gamma = \Sigma_\lambda\ (1/E\gamma)\ (dn_{\pi\lambda}/d\Omega)\ \sigma(a + \gamma \rightarrow b + c)$$

$\pi \rightarrow$ E or M
$\lambda \rightarrow 1,2,\ldots.$

$\Downarrow$

$$\sum_\lambda \Sigma(1/E\gamma)\ (dn_{\pi\lambda}/d\Omega)\ [(2j_b + 1)(2j_c + 1) / 2(2j_a + 1)]\ (k_{CM}{}^2 / k_\gamma{}^2)\ S(E)\ E\ \exp(-2\pi\eta)$$

S(E) can be extracted from the measured Coulomb excitation cross sections if

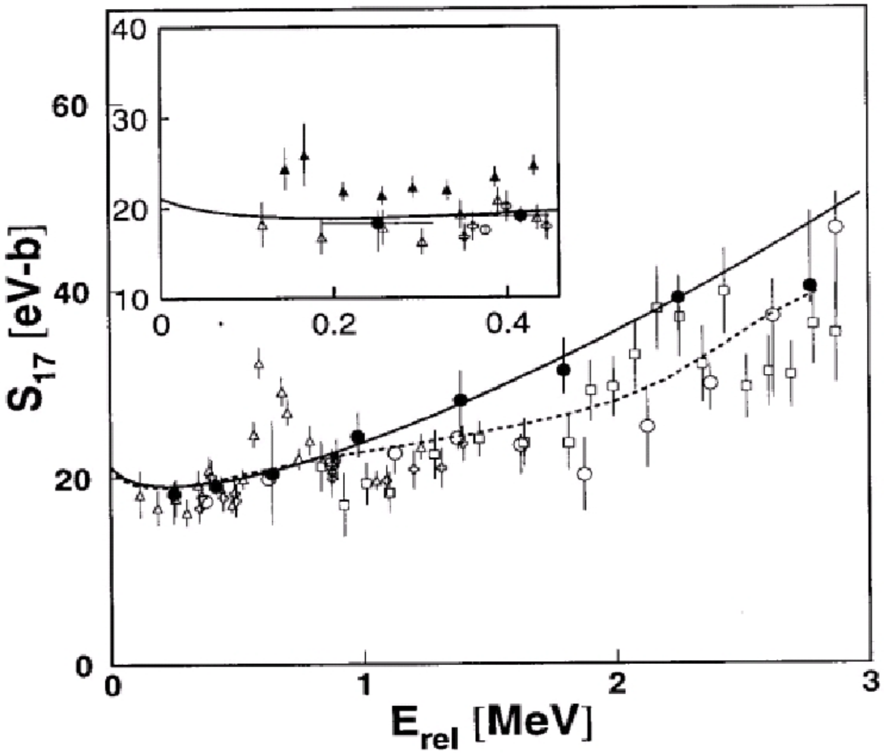
- **Projectile excitation is dominated by single multipolarity**
- **Application of the first order theory is of sufficient accuracy**
- **The point like projectile excitation is valid (we may have nuclei with large R)**
- **Influence of the strong nuclear field on the excitation process is negligible**

Applications of the Coulomb Dissociation Method

Radiative fusion reaction: $p + {}^7Be \rightarrow {}^8B + \gamma$

Relevant for solar neutrino problem. Determines the absolute values of the calculated ^8B ν flux.

WORLD DATA



Recent direct capture measurements

Hammache et al. PRL 80 (1998)

$p + {}^7Be$ $S_{17} = 18.5 \pm 2.4$ eV b 118 -186 keV

A.R. Junghans et al. PRL 88 (2003)

$p + {}^7Be$ $S_{17} = 22.3 \pm 1.2$ eV b 186-1200 keV

L.T. Baby et al. PRL 90 (2003)

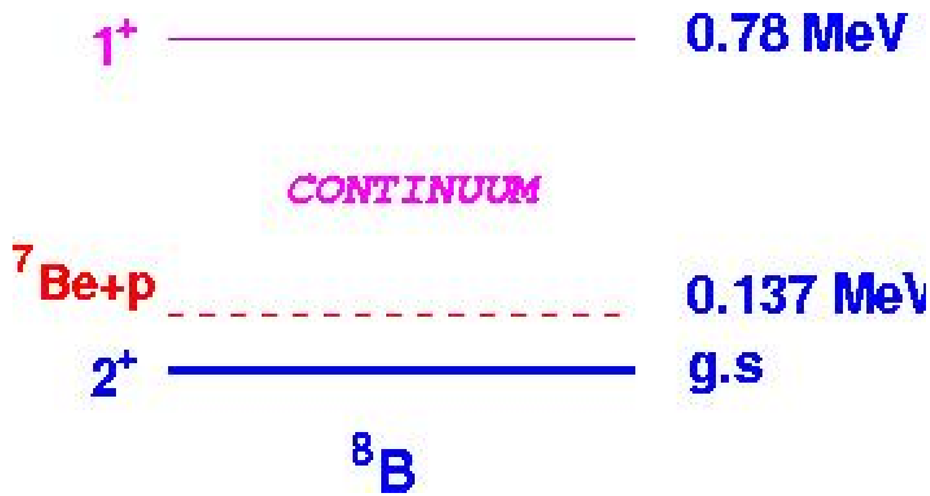
$p + {}^7Be$ $S_{17} = 21.2 \pm 0.7$ eV b 302-1078 keV

Determination from alternative approaches will Useful to resolve this difference

"^{8}B plays a crucial role in the interpretation of SNO experiments. Unfortunately the predicted value of ^{8}B flux normalization is quite uncertain, mainly due to the poorly known nuclear cross sections at low energies." , V. Bargerner, D. Marfatia and K. Whisnant, PRL 88 (2002).

Coulomb Dissociation of ^{8}B

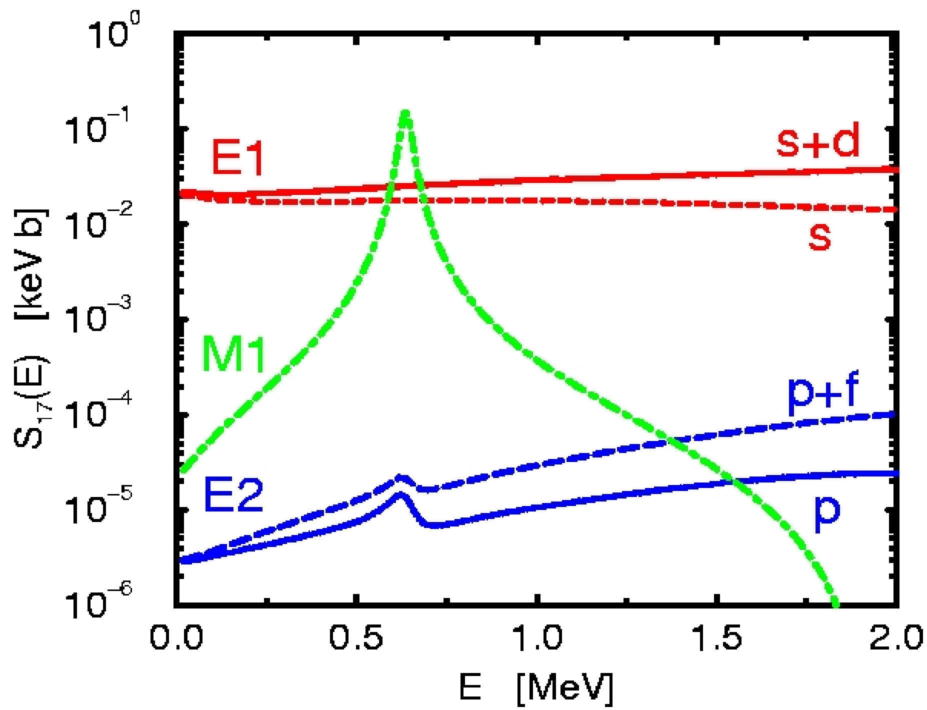
Determine the rate of the reaction $p + {}^{7}Be \rightarrow {}^{8}B + \gamma$ from the Coulomb dissociation of ^{8}B on a heavy target.



$$^{8}B + {}^{208}Pb \rightarrow {}^{8}B^* + {}^{208}Pb$$
$$\quad\quad\quad\quad\quad\quad\rfloor\; p + {}^{7}Be$$

In the Coulomb excitation of ^{8}B E1, E2, and M1 multipoles can contribute

In direct capture calculations within a single-particle model with Woods-Saxon potential , E1 multipolarity dominates



Nuclear interactions effects

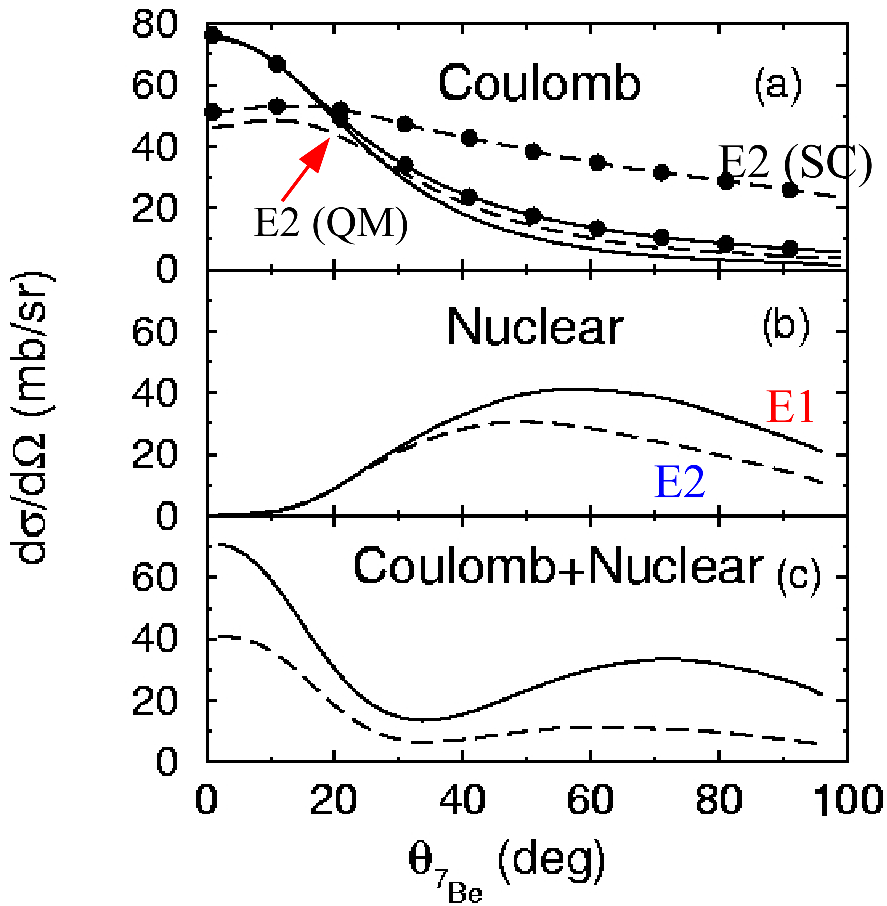
^{8}B may have an extended size QM calculations.

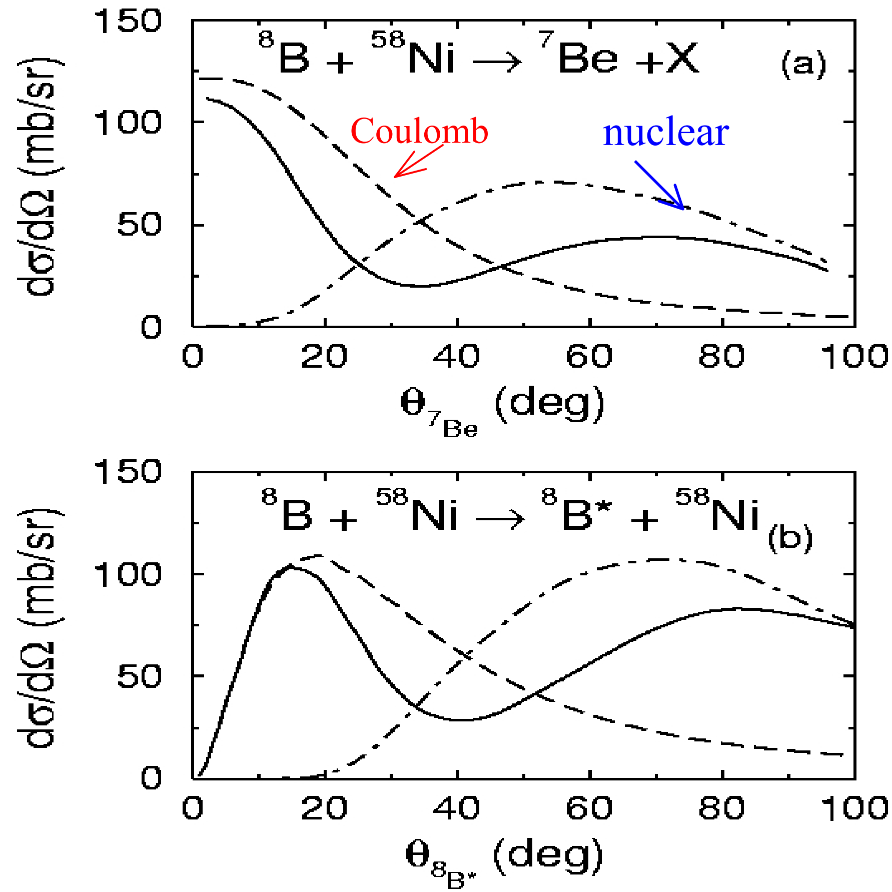
Coulomb Dissociation of ^8B

Shyam, Thompson, PRC 59(1999)

Experiment – I: University of Notre Dame, J von Schwarzenberg, PRC53 (1996)

Reaction: $^8B + ^{58}Ni \rightarrow ^7Be + ^{58}Ni$ E = 25.8





Semiclassical approximation is not valid for $\geq 20^0$

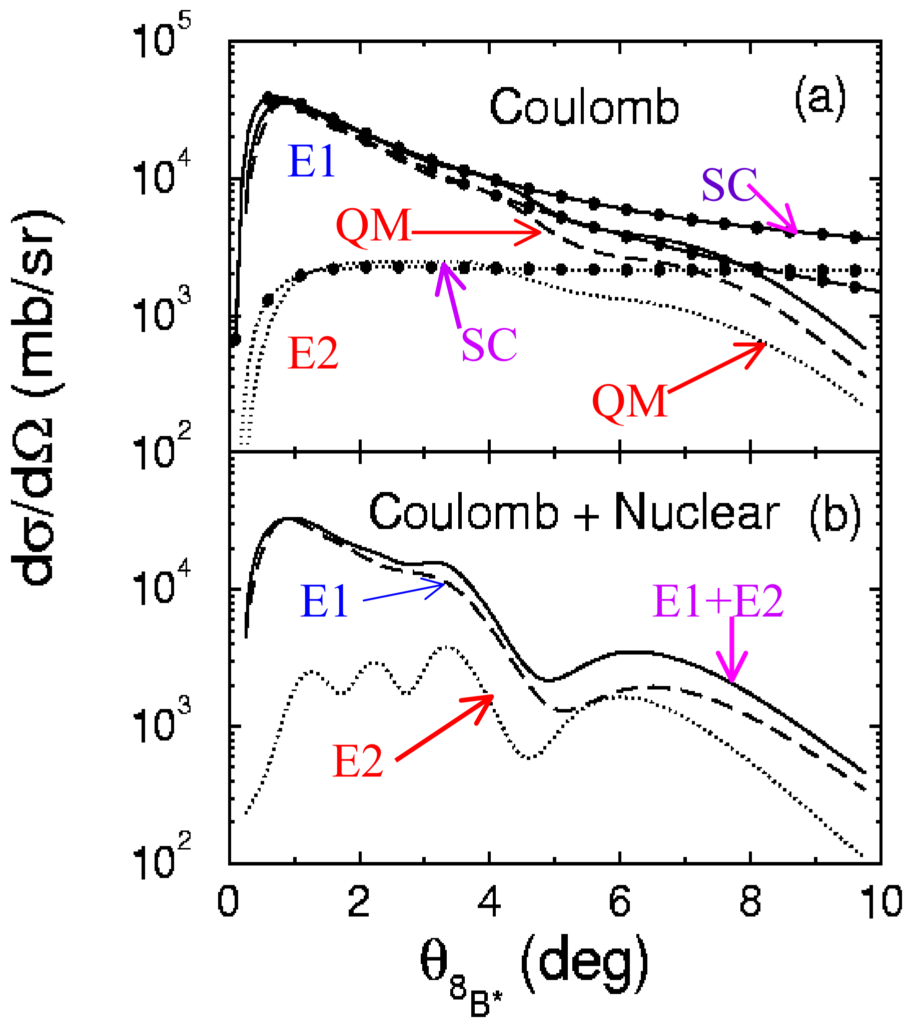
E2 and nuclear breakup Effects are quite large

Angular distributions of ^{7}**Be and ^{8}B* are not the same**

Coulomb Dissociation of ^{8}B

Experiment –II: RIKEN, Japan, T. Kikuchi et al., Phys. Lett B391(1997)

Reaction: $^{8}B + {}^{208}Pb \rightarrow {}^{8}B^{*}\ ({}^{7}Be\text{-}p) + {}^{208}Pb$, E = 51.2 MeV/nucleon



For $\theta_{8B^*} \leq 4$ deg, conditions for the applicability of the CD method are satisfied.

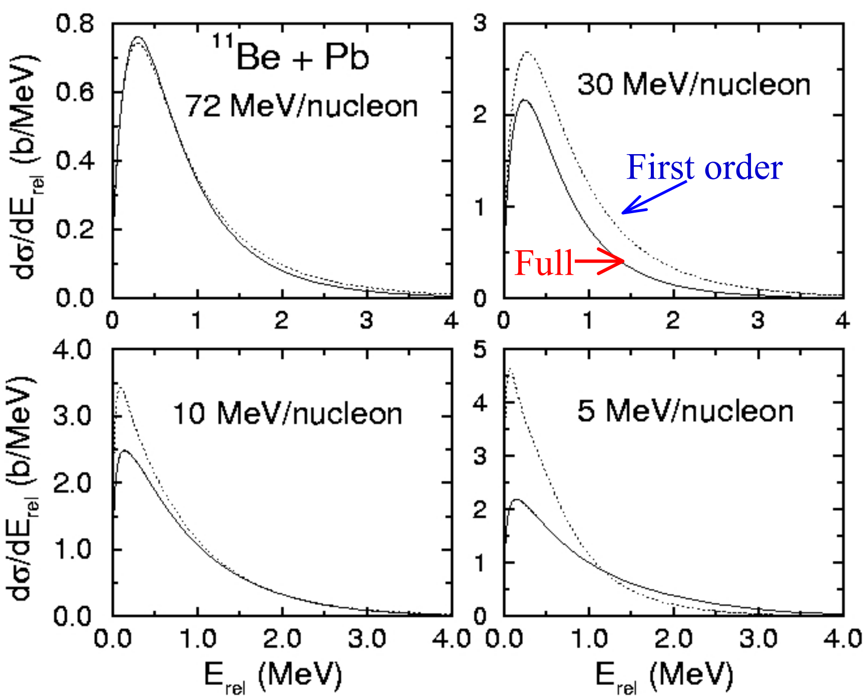
E1 mulipolarity dominates
E2 and nuclear excitation effects are negligible.
Semiclassical approximation is valid.

Data in this regime can be used for the extraction of S-factor using the SC theory.

Shyam, Thompson, PRC 59 (1999)
Banerjee, Shyam, PRC 62 (2000)

Role of postacceleration effects

In a reaction a + A → b + c + A,
$Z_b \neq Z_c$ then $E_b \geq E_b$(allowed by mass ratio).
Affects the rel. energy spectrum of the fragments.
This Effect is not included in the first order theory.

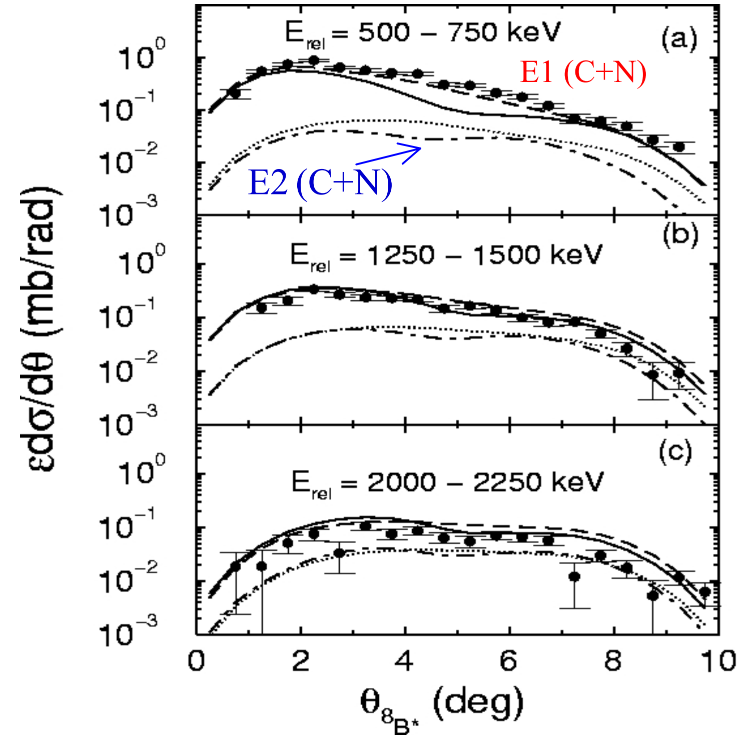


Postacceleration effects not important at higher Beam Energies.

Banerjee et al., PRC 65 (2003)

Extraction of S_{17} factor

$^8B + ^{208}Pb \rightarrow ^8B^* (^7Be\text{-}p) + ^{208}Pb$, E = 51.2 MeV/nuc



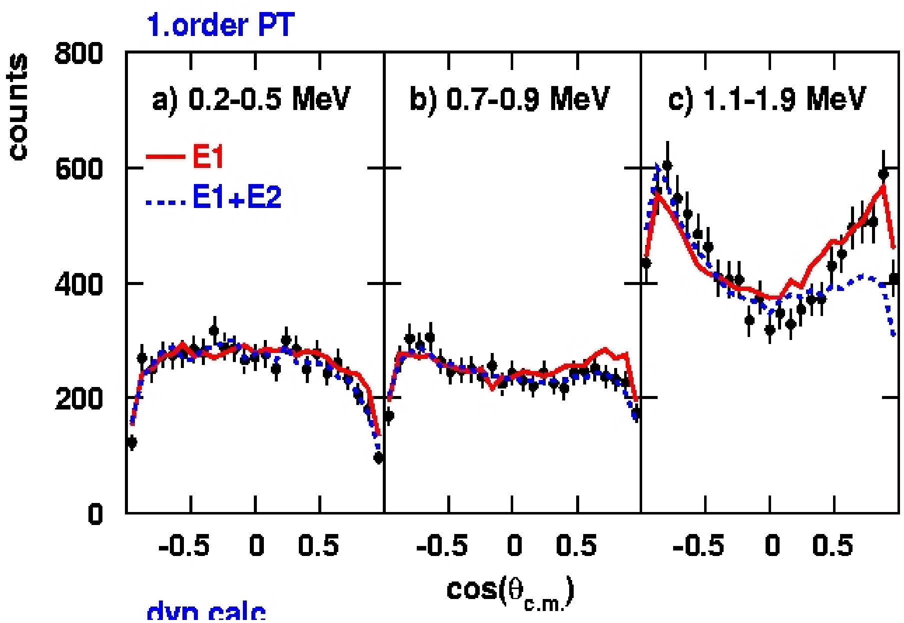
E_{rel} = 500-750 keV is suitable for the application of the CD method

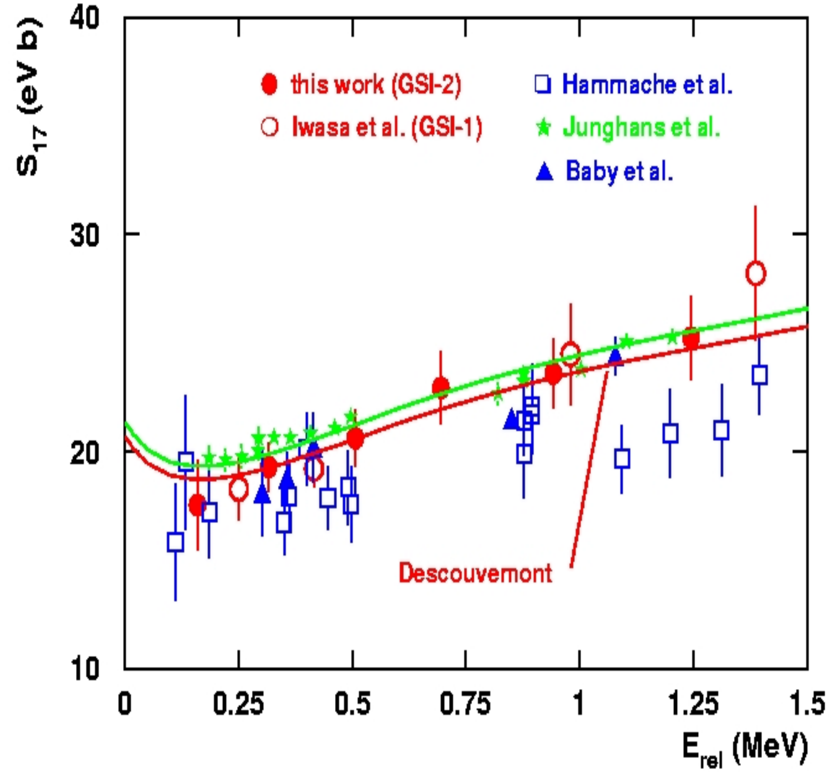
S_{17} = 18.2 eV b

Coulomb Dissociation of ^{8}B

Experiment –III: GSI, Germany, , F. Schuemann PRL 90 (2003), PRC (2006)

Reaction: $^{8}B + ^{208}Pb \rightarrow ^{8}B^{*}\ (^{7}Be\text{-}p) + ^{208}Pb$, E ; 250 MeV/nucleon





S_{17} = 20.6 ± 2.0 eV b

Latest CD S_{17} results are in good agreement with those obtained in the direct (p,γ) measurements by Junghans et al.

Also slopes of the CD and direct capture measurements are in agreement

Other Applications of the Coulomb Dissociation Method

- **Coulomb dissociation of ^{9}Li for determining the rate of the ^{8}Li (n,γ) ^{9}Li reaction**

After the production of ^{7}Li (big bang nucleosynthesis) the synthesis of ^{12}C follows the chain

^{7}Li (n,γ) ^{8}Li (α,n) ^{11}B(n,γ) ^{12}B (β^{-}) ^{12}C

$\downarrow$

^{8}Li (n,γ) ^{9}Li (β^{-}, ν) ^{9}Be (p,α) ^{6}Li $\Rightarrow$ **Reaction flows back to lighter elements**

Preliminary study at Michigan State University, but detailed work is needed.

- **Coulomb dissociation of ^{15}C for determining the rate of the ^{14}C (n,γ) ^{15}C reaction**

Neutrons produced in the burning zone of the 1.3M AGB stars by the ^{13}C $(\alpha$,n) reaction
Can drive a CNO cycle in which an α particle is synthesized from 4 neutrons

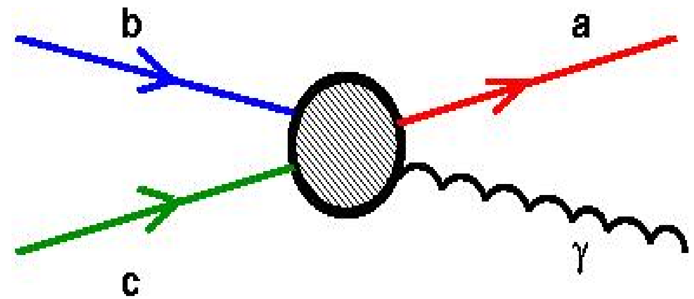
^{14}C (n,γ) ^{15}C (β^{-}) ^{15}N(n,γ) ^{16}N (β^{-}) ^{16}O (n,α) ^{14}C $\Rightarrow$ **Controlling reaction is ^{14}C (n,γ) ^{15}C**

- **Coulomb dissociation of ^{23}Al to study the stellar reaction ^{22}Mg (p,γ) ^{23}Al**

T. Gomi et al. Nucl. Phys. A758 (2005), and planned at GSI, Darmstadt

THE ANC METHOD

Direct capture reaction $b + c \rightarrow a + \gamma$



$$\sigma \propto |M|^2$$

$$M = \langle \varphi_a(\xi_b, \xi_b, r_{bc}) | O(r_{bc}) | \varphi_b(\xi_b) \varphi_c(\xi_c) \psi_i(r_{bc}) \rangle$$

$$I_{bc}^{A}(r_{bc}) = \langle \varphi_a(\xi_b, \xi_b, r_{bc}) | \varphi_b(\xi_b) \varphi_c(\xi_c) \rangle$$

$$= C_{\lambda j} f_{\lambda j} Y_{\lambda m}(\Omega)$$

$$r_{bc} \,?\, R_N, \quad f(r_{bc}) = C_{\lambda j} W_{\lambda+1/2}(2kr_{bc})/r_{bc}$$

At low energies $\psi_i(r_{bc})$ is given by Coulomb wave functions. So if the reaction is peripheral then the capture cross section is determined solely By the asymptotic normalization constant $C_{\lambda j}$. $\Rightarrow$ ANC

Methods for the determination of ANC

•Single particle potential model for a (b+c)

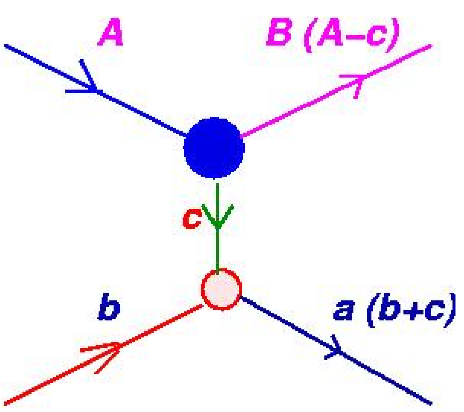
Assume b+c are bound together by a potential having a Woods-Saxon form.
Its depth is adjusted to reproduce the properties of the bound state.

The corresponding single particle wave function is $u_{\lambda j}$.

$$f(r_{bc}) = S_{\lambda j}^{1/2}\, u_{\lambda j}(r_{bc}) = \underline{S_{\lambda j}^{1/2}\, b_{\lambda j}}\, W_{\lambda+1/2}(2kr_{bc})$$

$\Downarrow$ Spectroscopic factor

$\downarrow$ $C_{\lambda j}$



From the transfer reaction b (A,B) a

$$d\sigma/d\Omega = |\langle \chi_{a\text{-}B}\, \phi_B \phi_a\, | V_{B\text{-}c} | \phi_A \phi_b\, \chi_{b\text{-}A} \rangle|^2$$

$$= |\langle \chi_{a\text{-}B}\, I_{ba}\, \phi_B\, | V_{B\text{-}c} | \phi_A\, \chi_{b\text{-}A} \rangle|^2$$

$$|I_{ba}|^2 = S_{\lambda j} |u_{\lambda j}|^2 = S_{\lambda j} b_{\lambda j} |W_{\lambda+1/2}|^2$$ If the transfer is peripheral

χ_s are the distorted waves in the initial and final channels

ANC from Transfer Reactions

Conditions to be satisfied

- Transfer reaction must be peripheral
- Single step transfer mechanism must dominate
- Compound nuclear contribution should be negligible
- Optical model potentials must be known with great accuracy

Applications to $p + {}^{7}Be \rightarrow {}^{8}B + \gamma$ Reaction

Experiment –I, Texas A & M group, Tribble et al. PRC 60 (1999), PRL 82 (1999)

${}^{10}B({}^{7}Be,{}^{8}B){}^{9}Be$, ${}^{14}N({}^{7}Be,{}^{8}B){}^{13}C$ transfer reactions with ${}^{7}Be$ beam

Elastic scattering cross section for the ${}^{10}B + {}^{7}Be$ and ${}^{14}N + {}^{7}Be$ were also measured

Peripheral nature of transfer process confirmed, but final channel OMP are unknown

ANC approximation was used for both (${}^{7}Be,{}^{8}B$) and (A,B) vertices.

$S_{17} = 16.6 \pm 1.9$ eV b, $S_{17} = 17.8 \pm 2.8$ eV b