The *mass* of small things

2nd Professor M.K. Banerjee Memorial Lecture

Saha Institute of Nuclear Physics Alumni Association

September 19, 2019

Amitava Raychaudhuri

University of Calcutta

Professor M.K. Banerjee

25.05.1930 - 18.02.2006

- Widely acclaimed physicist
- Interest spanning several fields
- Kept up with and contributed to the cutting edge
- Early computational work
- Development of institutions
- Taking interest in the younger colleagues.

Research spectrum:

Designing a beta-ray spectrometer (1953).

Nuclear resonance lineshape (1954)

Direct interaction theory of inelastic nuclear reactions. (1957-58)

Structure: Shell model calculations, SU(3) (1963-69)

Brückner-Hartree-Fock structure calculations (1969-72)

Pion-nucleon and pion-nucleus scattering (1972-83)

Chiral soliton model (1984-86)

Vacuum instability, proton spin, Chiral perturbation theory, instantons, ...

Professor M.K. Banerjee

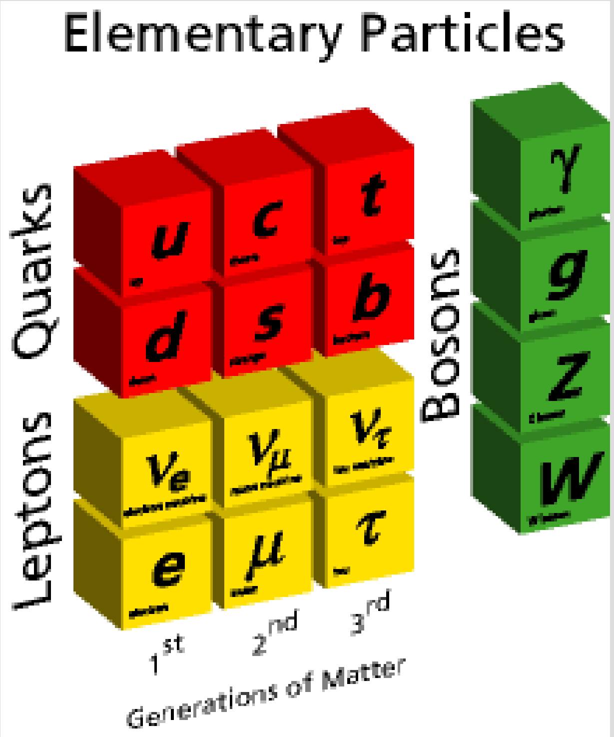
My interactions as a graduate student at Maryland

- Manoj Banerjee (not Manoj-*da*) was at Maryland from 1966.
- He was on sabbatical during 1973-74, when I joined
- Smiling face, always active, busy person
- Could not muster the courage to start a conversation.
- His Ph.D. student J. Barry Cammarata (who too was totally taken up with his calculations) was my contemporary
- Offered a course on `Field theory applications in many body problems' in my fourth year for which I registered
- Used to come to classical music programmes arranged by the Indian Students' Association.

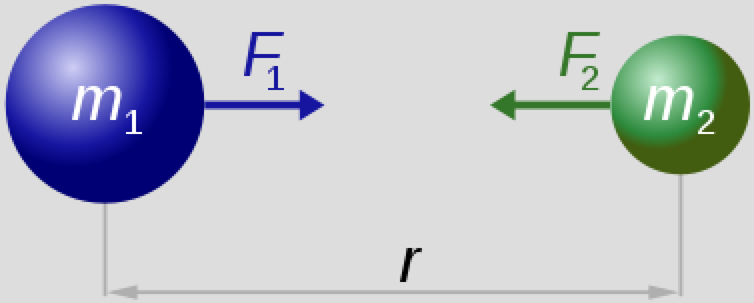
1981: Considering Directorship of SINP

Plan

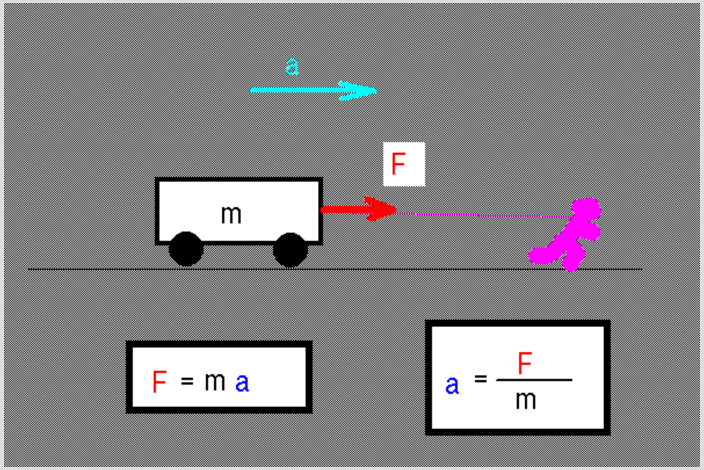
- Mass of an elementary particle
- Interactions, symmetries
- Fermion mass
- Higgs mechanism
- Standard Model
- Neutrino mass
- Majorana fermion, Majorana mass
- See-saw mechanism
- One loop masses
- Dark Matter, Dark Energy
- Conclusions



What is mass?



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$



Mass is responsible for the gravitational force.

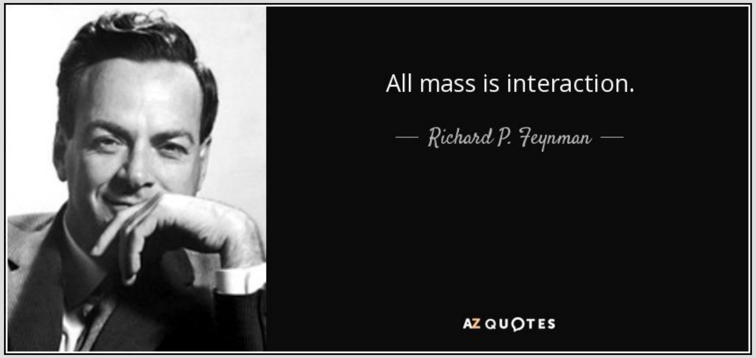
Mass tells us how a body responds to a force on it.

Motion under gravity is independent of the nature of the particle

Einstein: Gravity is due to spacetime curvature

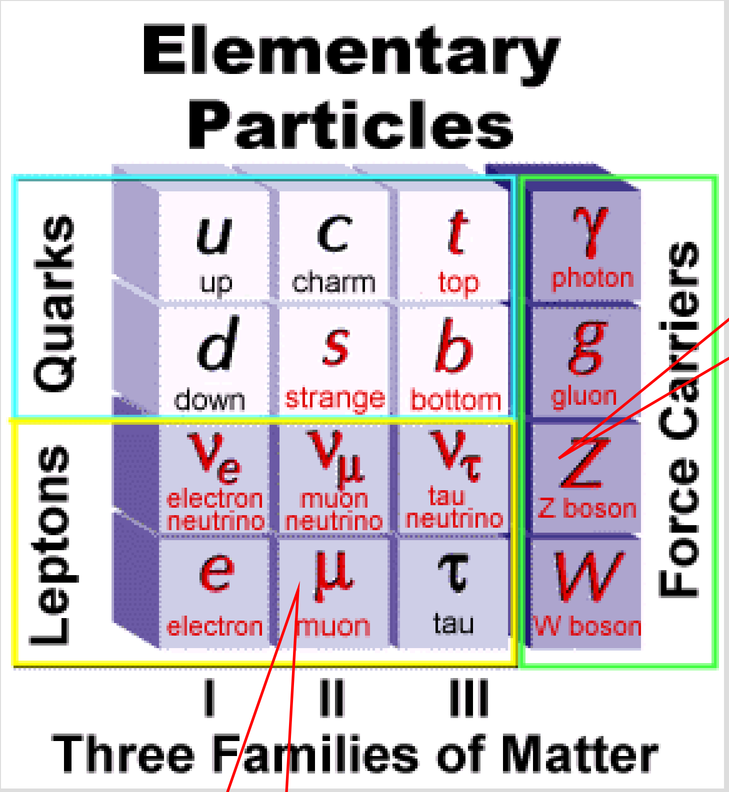
Origin of Mass?

- Gravitational interactions are due to the mass of a particle
- But what is the origin of mass?
- Mass is due to interactions



- Mass is a measure of inertia. More interactions make it difficult for a particle to move. Mass increases with interactions.

Elementary particles



Spin1 Bosons

Higgs Boson!

Spin 0 boson

Spin ½ Fermions

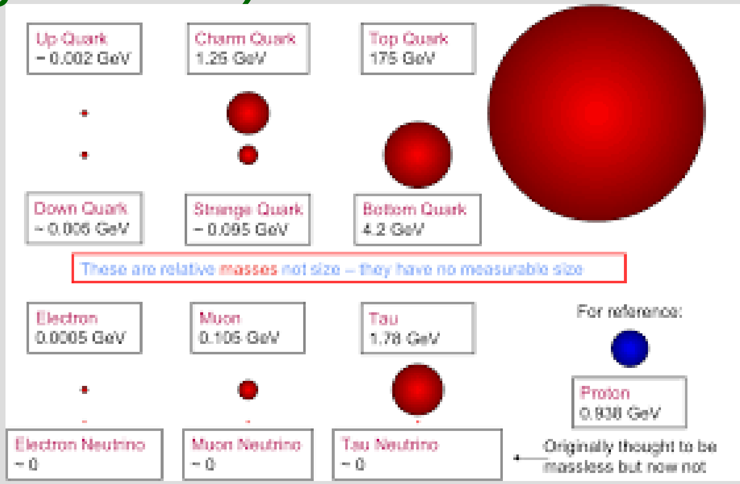
Source: http://electron9.phys.utk.edu/phys250/modules/module6/images/simplemodel2.gif

Elementary particle mass

- The mass of a particle is given by the quadratic term in the Lagrangian

- Scalar particles: no spin (e.g., Higgs boson)

$$L = m^2 \phi^* \phi$$



- Spin ½ fermions (electron)

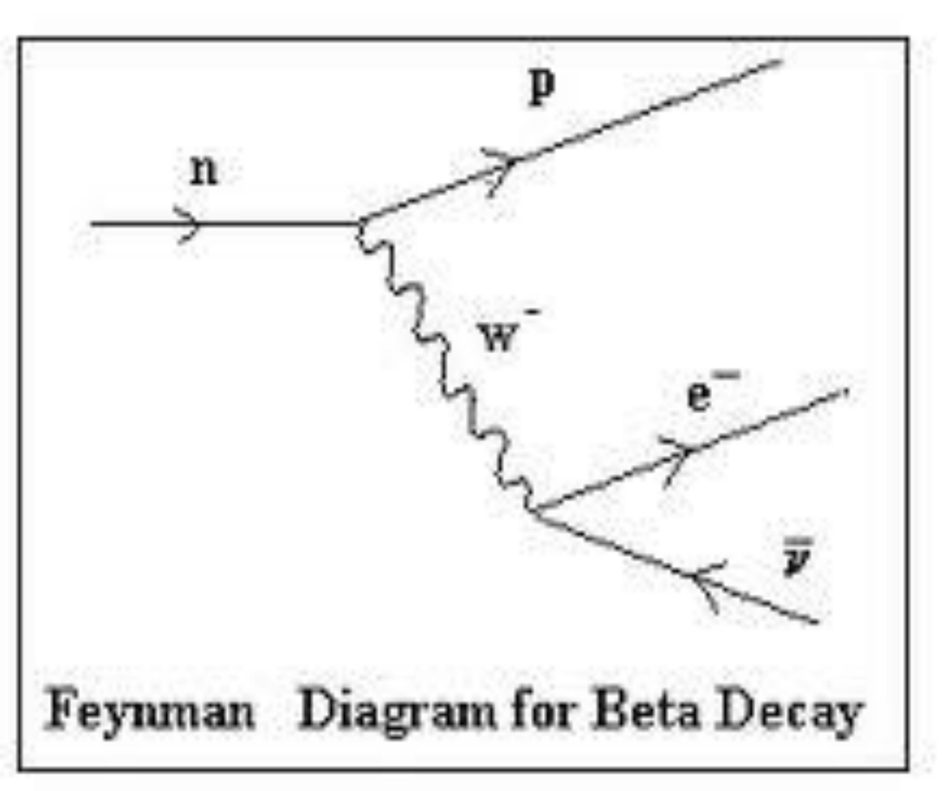
$$L = m \bar{\psi} \psi$$

- Spin 1 bosons (W-boson of weak interaction)

$$L = m^2 W_\mu W^\mu$$

Weak interactions

- Beta decay is a weak interaction
- One nucleus changes to another $\Rightarrow$ n $\rightarrow$ p e $\bar{\nu}$
- Here the exchanged particle is a W-boson
- The W-boson is heavy $m_W \sim 80\ m_p$



Massive spin -1 mediating particle

- Massless spin – 1 force mediators are familiar: The photon
- Can be readily included in quantum mechanics and quantum field theory (QED)
- Yang & Mills, Shaw, Utiyama, Schwinger, extended it to apply to strong and weak interactions

- In QED perturbation theory calculations yield testable corrections: e.g., Lamb shift
- Fails for massive spin – 1 particle (divergences!!)

The solution lies in the Brout-Englert-Higgs mechanism

Forces, Potential

- Electrostatic force $\underline{F} = e\,\underline{E}$

  Electric field $\underline{E} = -\underline{\nabla}\phi$, ϕ is the electrostatic potential.

- Lorentz force $\underline{F} = e\,(\underline{v} \times \underline{B})$

  Magnetic field $\underline{B} = \underline{\nabla} \times \underline{A}$, $\underline{A}$ is known as the vector potential.

Gauge symmetry

Since $\underline{\nabla} \times (\underline{\nabla}\theta) = 0$

If $\underline{A} \rightarrow \underline{A} + \underline{\nabla}\theta \Rightarrow \underline{B} \rightarrow \underline{B}$ *(A symmetry!)*

Symmetry

- Symmetries of the laws of physics $\Rightarrow$ e.g., symmetry under translations or rotations.
- Symmetry implies conservation laws $\Rightarrow$ Rotation symmetry results in conservation of angular momentum

Symmetries $\Rightarrow$ Interactions

Quantum Mechanics: Freedom to make *phase transformations* is a symmetry.

These are also sometimes called gauge symmetries.

Such symmetries are related to interactions.

These are the Gauge Theories.

Symmetry and Interactions

- In QM the state of a system is specified by a function often denoted by $\psi(x)$ – the wave function.
- At every point x, the wave function gives a complex number. So, it is a *complex function* of x.

Prob. of finding particle between x *and* $x + dx = |\psi(x)|^2 dx$

Momentum depends on the derivative: $-i\hbar\psi(x)^* \underline{\nabla}\,\psi(x)$

So, if $\psi(x) \rightarrow exp[i\theta]\ \psi(x)$ then $\psi(x)^* \rightarrow exp[-i\theta]\ \psi(x)^*$

If θ independent of x, it does not affect physics

as the $\psi(x)^{*}\cdots\ \psi(x)$ combination is unchanged

It is a symmetry, a <u>phase</u> symmetry!

Symmetry and Interactions (Contd.)

What if $\psi(x) \rightarrow \exp[i\theta(x)]\,\psi(x)$, with θ varying from point to point?

Prob. at $x = |\psi(x)|^2$ is unchanged

Momentum $-i\hbar\psi(x)^* \underline{\nabla}\,\psi(x)$ is affected due to an extra term $\sim [\underline{\nabla}\theta(x)]\,\psi(x)$.

So, it does not seem to work for $\underline{p}$. However, recall:

$\underline{p} + e\,\underline{A}/c$: the momentum in the presence of an em field

If $\underline{A} \rightarrow \underline{A} + \underline{\nabla}\theta \Rightarrow \underline{B} \rightarrow \underline{B}$ *(A symmetry!)*

Symmetry and Interactions (Contd.)

If simultaneously $\psi(x) \rightarrow exp[ie\theta\,(x)]\,\psi(x)$ and
$\underline{A} \rightarrow \underline{A} + \underline{\nabla}\theta$ then the symmetry can be ensured

The extra θ-dependent piece in the momentum from one term may be compensated by that due to the other.

In 4-d, the ordinary derivative ∂_μ is replaced by $(\partial_\mu - ie\, A_\mu)$

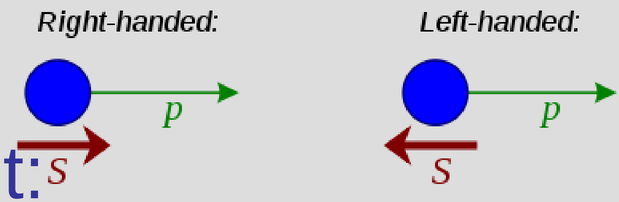
Thus if *local* gauge transformation is to be a symmetry electromagnetism must be there.
Note $\underline{A} \cdot \underline{A}$ *(or generalising* $A_\mu A^\mu$*) not gauge invariant.*

Photon mass $m^2 A_\mu A^\mu$ *not allowed by the symmetry!*

Fermion mass

- Fermions have spin! Left- and right-handed fermions:

$$\psi = \psi_L + \psi_R$$



- Fermion mass couples left to right:

$$m\,\bar{\psi}\,\psi = m(\bar{\psi}_R\,\psi_L + \bar{\psi}_L\,\psi_R)$$

$$\psi_L \leftrightarrow \psi_R \quad \text{under parity}$$

- The mass term can be included in the Lagrangian if it is a singlet under the symmetries
- If there is only left-handed (or right-handed) component then m=0.

Gauge theories: interactions

- We have a good understanding of basic particles and forces that act between them

- Strong, Electromagnetic, and Weak interactions all can be neatly framed by <u>gauge symmetry</u> principles.
- Which makes things calculable and predictions possible.

- Except ….. The symmetry which does this has a price: All particles must be massless!!

Glashow-Salam-Weinberg Model: fermions

- Electroweak interactions only
- Symmetry group: $SU(2)_L \times U(1)$ Two gauge couplings: g and g'
- Four gauge bosons $W^{\pm}_{\mu}, W^{3}_{\mu}, B_{\mu}$ *the last two* neutral

$Q = T_3 + Y/2$

- Leptons $\Psi_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$ (Y =-1), e_R (Y= -2)

Lagrangian $$\mathcal{L} = \bar{\Psi}_L[\partial^\mu - \frac{g}{2}W^{\mu a}\tau^a - \frac{g'}{2}(-1)B^\mu]\gamma_\mu\Psi_L + \bar{e}_R[\partial^\mu - \frac{g'}{2}(-2)B^\mu]\gamma_\mu e_R$$

Y Y

- Quarks $\begin{pmatrix} u \\ d \end{pmatrix}_L$ (Y=+1/3) , u_R (Y=4/3) , d_R (Y = -2/3)

- Fermions: SU(2) singlets have no $W^{\pm}$ interactions. Parity violation!
- Left-handed oublets, right-handed singlets ⇒ Mass term is non-singlet, not allowed

The Higgs mechanism

- **The symmetry requires that all elementary particles have zero mass.**
- **Quarks, leptons, W-bosons, photons all massless!**
- **That contradicts experiments!**

- **One ingredient was missing.**

- **The Higgs field. It is the entity which breaks the symmetry, but subtly! Preserves the good features of the symmetry and yet makes particles massive.**
- **Higgs boson is a wave in this field. Higgs bosons have been detected at CERN.**

Higgs' paper

Vacuum is not empty!

It is filled with the Higgs field

VOLUME 13, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson[3] has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

The simplest theory which exhibits this behavior is a gauge-invariant version of a model used by Goldstone[2] himself: Two real[4] scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density

$$L = -\tfrac{1}{2}(\nabla\varphi_1)^2 - \tfrac{1}{2}(\nabla\varphi_2)^2 - V(\varphi_1^{\,2}+\varphi_2^{\,2}) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad (1)$$

where

$$\nabla_\mu\varphi_1 = \partial_\mu\varphi_1 - eA_\mu\varphi_2,$$

$$\nabla_\mu\varphi_2 = \partial_\mu\varphi_2 + eA_\mu\varphi_1,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

e is a dimensionless coupling constant, and the metric is taken as $-+++$. L is invariant under simultaneous gauge transformations of the first kind on $\varphi_1 \pm i\varphi_2$ and of the second kind on A_μ. Let us suppose that $V'(\varphi_0^{\,2}) = 0$, $V''(\varphi_0^{\,2}) > 0$; then spontaneous breakdown of U(1) symmetry occurs. Consider the equations [derived from (1) by treating $\Delta\varphi_1$, $\Delta\varphi_2$, and A_μ as small quantities] governing the propagation of small oscillations about the "vacuum" solution $\varphi_1(x) = 0$, $\varphi_2(x) = \varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\} = 0, \qquad (2a)$$

$$\{\partial^2 - 4\varphi_0^{\,2}V''(\varphi_0^{\,2})\}(\Delta\varphi_2) = 0, \qquad (2b)$$

$$\partial_\nu F^{\mu\nu} = e\varphi_0\{\partial^\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\}. \qquad (2c)$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^{\,2})\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_\mu = A_\mu - (e\varphi_0)^{-1}\partial_\mu(\Delta\varphi_1),$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu}, \qquad (3)$$

into the form

$$\partial_\mu B^\mu = 0, \quad \partial_\nu G^{\mu\nu} + e^2\varphi_0^{\,2}B^\mu = 0. \qquad (4)$$

Equation (4) describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e = 0$) the situation is quite different: Equations (2a) and (2c) describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of (2c) is just the linear approximation to the conserved current: It is linear in the vector potential, gauge invariance being maintained by the presence of the gradient term.[5]

When one considers theoretical models in which spontaneous breakdown of symmetry under a semisimple group occurs, one encounters a variety of possible situations corresponding to the various distinct irreducible representations to which the scalar fields may belong; the gauge field always belongs to the adjoint representation.[6] The model of the most immediate interest is that in which the scalar fields form an octet under SU(3): Here one finds the possibility of two nonvanishing vacuum expectation values, which may be chosen to be the two $Y = 0$, $I_3 = 0$ members of the octet.[7] There are two massive scalar bosons with just these quantum numbers; the remaining six components of the scalar octet combine with the corresponding components of the gauge-field octet to describe

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massive vector bosons. There are two $I = \frac{1}{2}$ vector doublets, degenerate in mass between $Y = \pm 1$ but with an electromagnetic mass splitting between $I_3 = \pm\frac{1}{2}$, and the $I_3 = \pm 1$ components of a $Y = 0$, $I = 1$ triplet whose mass is entirely electromagnetic. The two $Y = 0$, $I = 0$ gauge fields remain massless: This is associated with the residual unbroken symmetry under the Abelian group generated by Y and I_3. It may be expected that when a further mechanism (presumably related to the weak interactions) is introduced in order to break Y conservation, one of these gauge fields will acquire mass, leaving the photon as the only massless vector particle. A detailed discussion of these questions will be presented elsewhere.

It is worth noting that an essential feature of the type of theory which has been described in this note is the prediction of incomplete multiplets of scalar and vector bosons.[8] It is to be expected that this feature will appear also in theories in which the symmetry-breaking scalar fields are not elementary dynamic variables but bilinear combinations of Fermi fields.[9]

[1] P. W. Higgs, to be published.

[2] J. Goldstone, Nuovo Cimento 19, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. 127, 965 (1962).

[3] P. W. Anderson, Phys. Rev. 130, 439 (1963).

[4] In the present note the model is discussed mainly in classical terms; nothing is proved about the quantized theory. It should be understood, therefore, that the conclusions which are presented concerning the masses of particles are conjectures based on the quantization of linearized classical field equations. However, essentially the same conclusions have been reached independently by F. Englert and R. Brout, Phys. Rev. Letters 13, 321 (1964): These authors discuss the same model quantum mechanically in lowest order perturbation theory about the self-consistent vacuum.

[5] In the theory of superconductivity such a term arises from collective excitations of the Fermi gas.

[6] See, for example, S. L. Glashow and M. Gell-Mann, Ann. Phys. (N.Y.) 15, 437 (1961).

[7] These are just the parameters which, if the scalar octet interacts with baryons and mesons, lead to the Gell-Mann–Okubo and electromagnetic mass splittings: See S. Coleman and S. L. Glashow, Phys. Rev. 134, B671 (1964).

[8] Tentative proposals that incomplete SU(3) octets of scalar particles exist have been made by a number of people. Such a rôle, as an isolated $Y = \pm 1$, $I = \frac{1}{2}$ state, was proposed for the κ meson (725 MeV) by Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 11, 42 (1963). More recently the possibility that the σ meson (385 MeV) may be the $Y = I = 0$ member of an incomplete octet has been considered by L. M. Brown, Phys. Rev. Letters 13, 42 (1964).

[9] In the theory of superconductivity the scalar fields are associated with fermion pairs; the doubly charged excitation responsible for the quantization of magnetic flux is then the surviving member of a U(1) doublet.

A classical field theory analysis of small oscillations

Higgs mechanism

In a crowded room passage is difficult

Movement is hindered, there is extra inertia

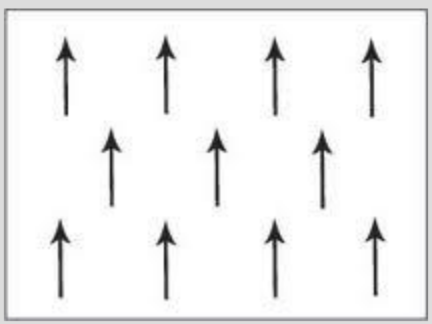
This idea is used in the Higgs mechanism to generate masses

Higgs mechanism

The vacuum of the universe is not empty. It has a background (condensate) of the Higgs field. This is responsible for particle masses.

Particle masses are proportional to their strength of coupling to the Higgs field.

Higgs boson



Higgs condensate

How can we know about the Higgs condensate?

Let’s return to the crowded room

What happens when a rumour reaches the room?

Higgs boson

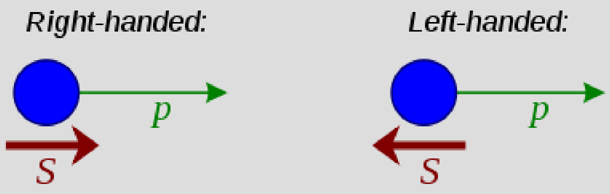
The rumour gets transmitted from person to person. The rumour travels like a wave in a medium.

A disturbance in the Higgs condensate is a Higgs boson. This is what has been detected (2012).

Standard Model

- The Standard Model describes strong and electroweak interactions.
- Mediated by gluons, W-boson, Z-boson, and photon.

- Parity violation: Asymmetry between left-handed and right-handed fermions.
- Mass-term not allowed
- Higgs mechanism to the rescue.



- Masses of W, Z , quarks and leptons via Higgs field

- No ν_R in SM $\Rightarrow$ Neutrino is massless. Chosen for consistency with information of that era.
- (B-L) is a symmetry of the Standard Model

Higgs boson mass: Naturalness

- The Higgs boson has been detected at the Large Hadron Collider with a mass ~ 125 GeV
- This is a clear vindication of the Brout-Englert-Higgs mechanism.

- In a theory with several energy scales, the mass of an elementary scalar particle is pushed to the highest scale.

- For various reasons, new theories predict other interactions emerging at higher scales.
- Even otherwise, the scale of Gravity is near 10^{19} GeV.
- Why is the Higgs scalar so light? Symmetry protection?
- Is the Higgs boson a composite particle?

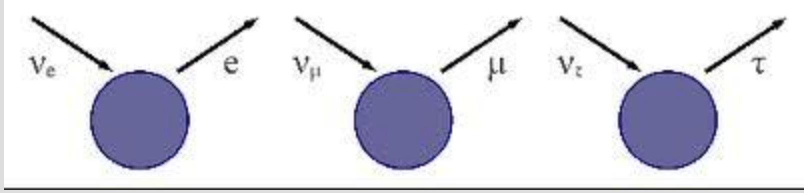
Neutrino mass

The dog that did not bark, W. Pauli 1933

`The Adventure of Silver Blaze',

A. Conan Doyle, 1892

Neutrino properties (contd.)



Three types: ν_e , ν_μ, ν_τ are known.

A ν_e is produced from an initial electron (e). Similarly, ν_μ, ν_τ are associated with μ, τ leptons.

Many properties discovered in the past two decades

e^- electron	**μ^-** muon	**τ^-** tau	$Q = -e$
ν_e electron neutrino	**ν_μ** muon neutrino	**ν_τ** tau neutrino	$Q = 0$
1st gen.	2nd gen.	3rd gen.	

Neutrino interactions

W$^{\pm}$ exchange

CC: Charge current

$\nu_e + n \rightarrow e + p$

Z exchange

NC: Neutral current

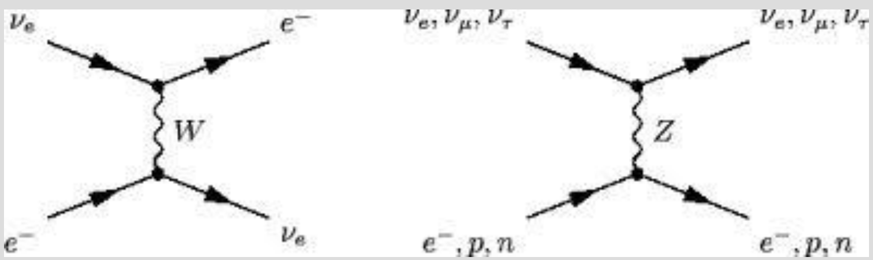
$\nu_x + p \rightarrow \nu_x + p$

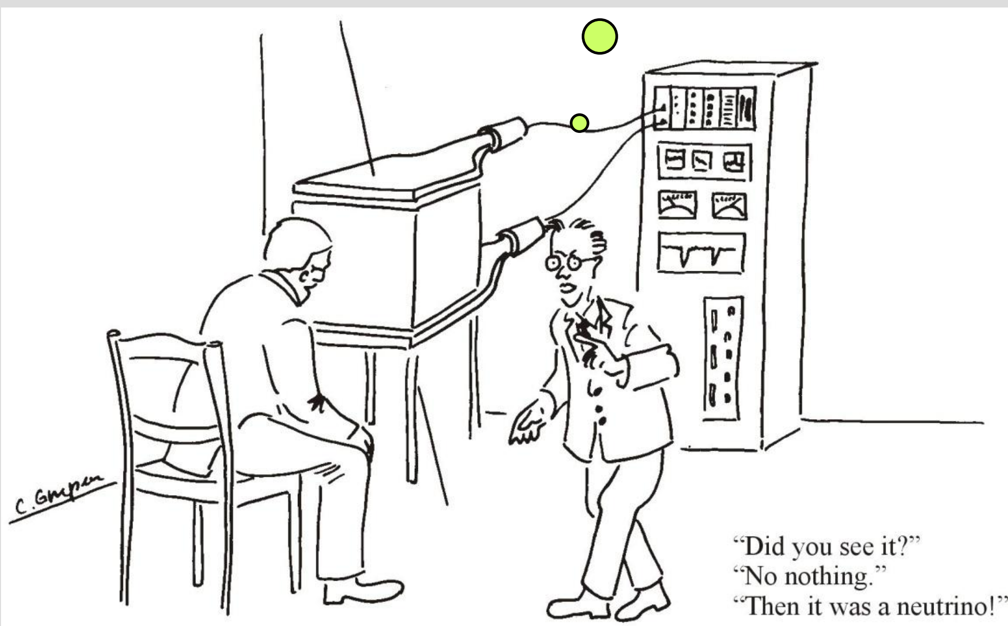
$\nu_x + n \rightarrow \nu_x + n$ $(x = e, \mu, \tau)$

Did you see it?

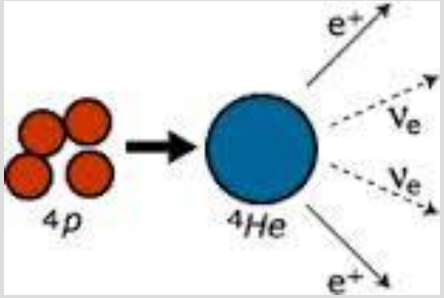
No, Nothing!

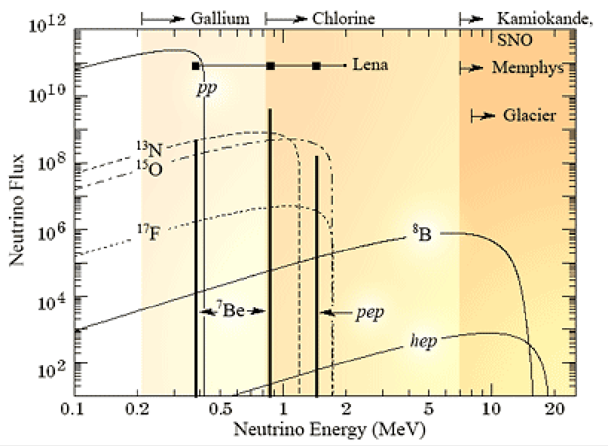
Then it was a neutrino





Solar neutrinos





- Sun generates heat and light through fusion reactions

  $4p \rightarrow {}^4He + 2\,e^+ + 2\,\nu_e + 27\text{ MeV}$ (i)
- Just like sunlight, solar neutrinos are reaching us (day & night!)

- Reaction (i) does not take place in one go. Rather, it is the consequence of a cycle of reactions, e.g.

  $p + p \rightarrow {}^2H + e^+ + \nu_e$ ← *pp neutrinos*

  The ν_e energy spectra from these reactions are well-known.

- Robust prediction of the number of solar neutrinos reaching the earth as a function of energy is possible. These have been detected by several expts. But …

Solar neutrino results

Expt	Obsvd/Predn	E_{th} (MeV)	Type
Homestake (from 1968)	0.335 ± 0.029	0.8	Radiochemical $\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e^-$ (CC)
GNO, SAGE, Gallex	0.584 ± 0.039	0.233	Radiochemical $\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e^-$ (CC)
K, SuperK (1989)	0.459 ± 0.017	5.0	Water Cerenkov $\nu_e + e \rightarrow \nu_e + e$ (CC + NC)
SNO CC	0.347 ± 0.027	6.75	Cerenkov $\nu_e + d \rightarrow p + p + e^-$ (CC)
SNO NC	1.008 ± 0.123	2.2	$\nu + d \rightarrow n + p + \nu$ (NC)

Ray Davis Jr

Nobel: 2002

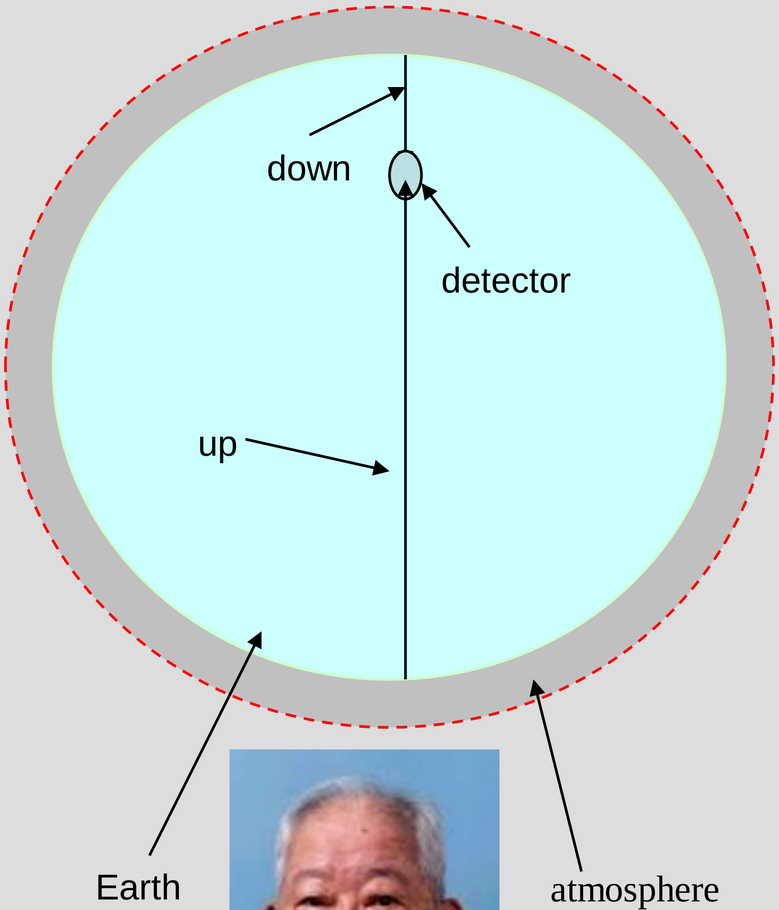
A.B. McDonald

Atmospheric neutrinos

Neutrinos are produced in the atmosphere from cosmic ray pion and kaon decays
e.g. $(\pi^- \rightarrow \mu^- + \overline{\nu}_\mu)$, $(\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu)$
and the charge conjugate processes
Typical energy ~ 1 GeV

Expectation: $R = (\# \, \nu_\mu + \overline{\nu}_\mu)/(\# \, \nu_e + \overline{\nu}_e) \approx 2$

SuperK: $R_{obs}/R_{mc} = 0.635 \pm 0.035 \pm 0.083$ (sub-GeV)
$= 0.604 \pm 0.065 \pm 0.065$ (multi-GeV)



T. Kajita

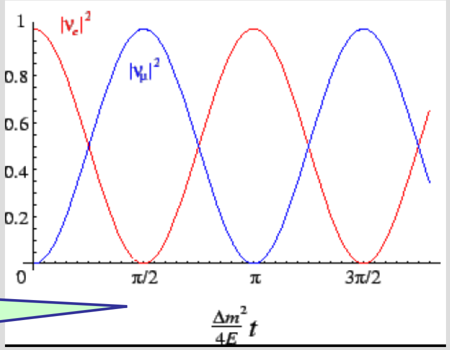
No. of ν_μ depends on zenith angle (up-down asymmetry)

No such effect for ν_e (1997)

Masatoshi Koshiba
Nobel: 2002

Neutrino oscillations

- A quantum mechanical phenomenon relying on the superposition principle.
- In the oscillation of a pendulum, the bob alternately reaches the left and right end-points of the trajectory.
- During travel, a ν_e becomes a ν_μ and then back again to a ν_e. This oscillation process continues.



$\text{Prob}(\nu_e \rightarrow \nu_\mu, L) = 4\, c^2\, s^2\, \sin^2(\pi L/\lambda)$

$c = \cos\theta$

$s = \sin\theta$

Maximal mixing

$\theta = \pi/4$

The oscillation wavelength (and hence probability!) depends on the neutrino energy. $\lambda = 4\pi E/\Delta$,
$\Delta = (m_2^2 - m_1^2)$

Some Quantum Mechanics

Stationary states: $H\,|\Psi_n\rangle = E_n\,|\Psi_n\rangle$

Time evolution: $|\Psi_n(t)\rangle = \exp(-iE_n t)\,|\Psi_n(0)\rangle$ (only a phase)

General state (t=0): $|\Psi(0)\rangle = \sum a_n\,|\Psi_n(0)\rangle$

General state (any t): $|\Psi(t)\rangle = \sum a_n \exp(-iE_n t)\,|\Psi_n(0)\rangle$

Phase differences $\sim (E_i - E_j)t$ → physics consequences

Neutrino stationary states: $|\nu_1\rangle, |\nu_2\rangle$

(mass eigenstates)

Neutrino flavour eigenstates: $|\nu_e\rangle, |\nu_\mu\rangle$

Mass ↔ Flavour states:

$$|\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta$$

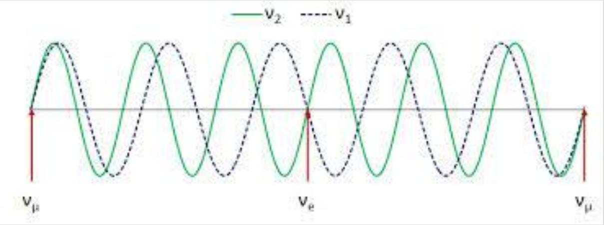
$$|\nu_\mu\rangle = -|\nu_1\rangle \sin\theta + |\nu_2\rangle \cos\theta$$

Quantum Mechanics of neutrino oscillations (contd.)

$|\nu_e\rangle$ produced at $t = 0 \rightarrow |\Psi(0)\rangle = |\nu_e\rangle = |\nu_1\rangle \cos\theta + |\nu_2\rangle \sin\theta$

At a later time: $|\Psi(t)\rangle = |\nu_1\rangle \cos\theta\, e^{-iE_1 t} + |\nu_2\rangle \sin\theta\, e^{-iE_2 t}$

$\text{Prob}(\nu_e \rightarrow \nu_\mu, L) = |\langle \nu_\mu|\Psi(t)\rangle|^2 = 4 c^2 s^2 |e^{-iE_1 t} - e^{-iE_2 t}|^2$



Neutrinos are ultra-relativistic: $p \gg m \Rightarrow E_i = (p^2 + m_i^2)^{1/2} \approx p + m_i^2/2p$

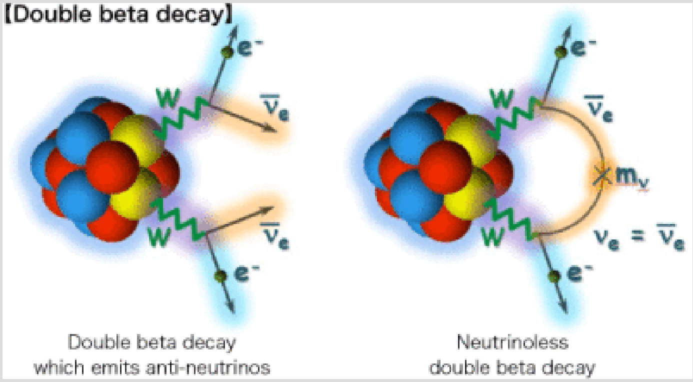
$$(E_1 - E_2)t = (m_1^2 - m_2^2)t/2p \equiv (\Delta/2p)t = \Delta L/2E$$

$\text{Prob}(\nu_e \rightarrow \nu_\mu, L) = 4 c^2 s^2 \sin^2(\pi L/\lambda)$ where $\lambda = 4\pi E/\Delta$

Survival Prob. = $\text{Prob}(\nu_e \rightarrow \nu_e, L) = 1 - \text{Prob}(\nu_e \rightarrow \nu_\mu, L)$

Majorana Neutrino?

- Can the neutrino be its own anti-particle? ($\nu \equiv \nu^c$?)
  The photon is its own anti-particle. (Also π^0)
- In such an event, lepton number is not conserved!
- Consequence $\Rightarrow$ Neutrino-less double beta decay ($0\nu2\beta$ process)
- Normal double beta decay ($2\nu2\beta$) : $X \rightarrow Y + 2\,e^- + 2\nu_e$
- Neutrino-less double beta decay ($0\nu2\beta$) : $X \rightarrow Y + 2\,e^-$ ($\propto \langle m_\nu \rangle^2$)
- Look for peak in $2e^-$ total energy
- Current limit $\langle m_\nu \rangle < 0.2$ eV.



Neutrino Masses

In SM no RH $\nu \Rightarrow$ No Dirac mass.
Also, Lepton No. is conserved $\Rightarrow$ No Majorana mass.

Dirac mass (Add right-handed neutrinos N_j)

N_j are singlets under SM (a sterile fermion)

$$\mathcal{L}_{Dirac\ mass} = -y^{\nu}_{ij} \bar{L}_i \tilde{H} N_j \ , \ \tilde{H} = i\sigma_2 H$$

Why is y^{ν}_{ij} so very small compared to Yukawa couplings for quarks and charged leptons?

Separate Higgs doublet with tiny vev for neutrinos?

Neutrino Masses

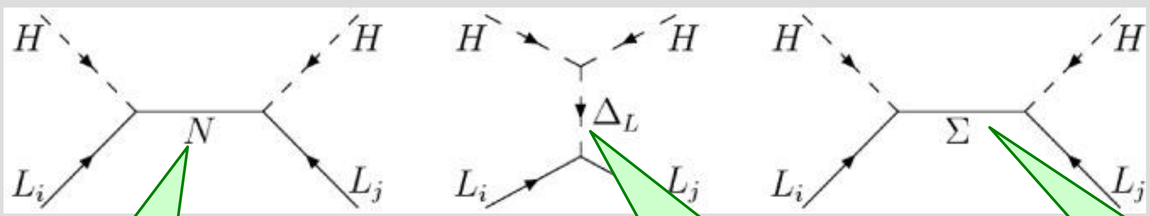
Majorana mass (Violate Lepton number)

⇒Leptogenesis?

e.g., Lepton No. violating interactions in BSM, R-parity violation in SUSY, etc.

$$\mathcal{L}_{Weinberg} = \frac{1}{2}\frac{1}{M_\nu} L_i L_j H H$$

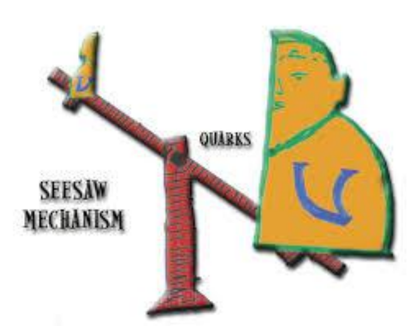
See-saw (Add new fermions/scalars)



SU(2) singlet

SU(2) triplet

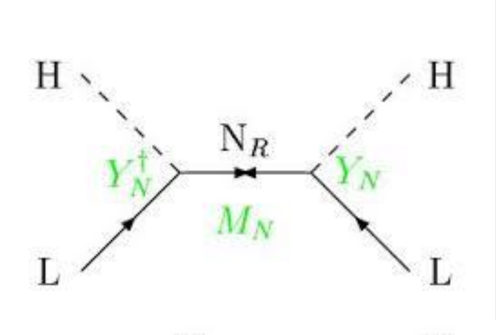
SU(2) triplet



Standard see-saws

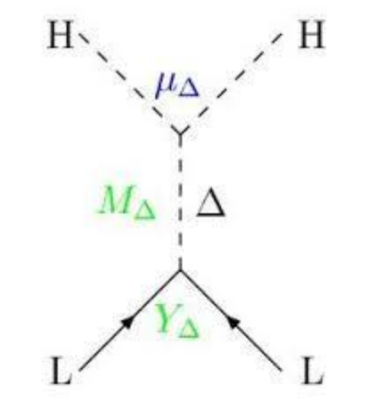
- **Type – I**

$$(m_\nu)_{ij} = v^2 \Sigma_k \left[\frac{(Y_N)^\dagger_{ik} (Y_N)_{kj}}{M_{N_k}} \right]$$



Majorana mass of N_k. Lepton No. violated.

- **Type – II** An SU(2) triplet scalar Δ. No RH ν
  Either Δ gets a vev violating lepton number
  or it has a trilinear coupling to a pair of H which get vev.

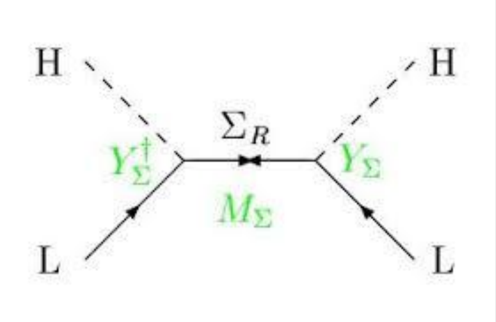


$$(m_\nu)_{ij} = (Y_\Delta)_{ij} \frac{\mu_\Delta v^2}{M_\Delta^2}$$

- **Type – III**
  Similar to Type – I but Σ_R are SU(2) triplets

(Three RH SU(2)-singlet neutrinos N_R /Σ_R to give masses to all neutrinos)

$$(m_\nu)_{ij} = v^2 \Sigma_k \left[\frac{(Y_\Sigma)^\dagger_{ik} (Y_\Sigma)_{kj}}{M_{\Sigma_k}} \right]$$



Majorana mass of Σ_k. Lepton No. violated.

Left-right symmetry

- Provides a natural room for N_R.
- Under $SU(2)_L$ x $SU(2)_R$ x $U(1)_{B-L}$ lepton multiplets: (2,1,1) and (1,2,1)
  $Q = T_{3L} + T_{3R} + (B-L)/2$

$$L_L = \begin{pmatrix} e_L \\ \nu_L \end{pmatrix} \quad L_R = \begin{pmatrix} e_R \\ N_R \end{pmatrix}$$

Type-I

- Different types of see-saw can be readily incorporated
- Choice of scalars:

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_2^+ \\ \phi_1^- & \phi_2^0 \end{pmatrix} \equiv (2,2,0)\ ,\ \Delta_L = \begin{pmatrix} \delta_L^{++} \\ \delta_L^{+} \\ \delta_L^{0} \end{pmatrix} \equiv (3,1,2)\ ,\ \Delta_R = \begin{pmatrix} \delta_R^{++} \\ \delta_R^{+} \\ \delta_R^{0} \end{pmatrix} \equiv (1,3,2)$$

Type-II

- Type-III see-saw, loop masses etc. possible

Massless neutrinos

J. Kersten and A. Yu. Smirnov, PRD 76, 073005 (2007)
R. Adhikari and A.R., PRD 84, 033002 (2011)

Type-I see-saw models which automatically lead to 3 massless Majorana neutrinos due to symmetries.

$$M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} . \text{ condition for zero eigenvalues of M}$$

$$M_D = \begin{pmatrix} x_1 & x_2 & x_3 \\ \alpha_1 x_1 & \alpha_2 x_2 & \alpha_3 x_3 \\ \beta_1 x_1 & \beta_2 x_2 & \beta_3 x_3 \end{pmatrix} \text{ and } M_R = \begin{pmatrix} M_1 & M_4 & M_5 \\ M_4 & M_2 & M_6 \\ M_5 & M_6 & M_3 \end{pmatrix}$$

1 zero eigenvalue $\Rightarrow$

$$\alpha_1 = \alpha_2 = \alpha_3 \text{ or } \beta_1 = \beta_2 = \beta_3$$
$$\text{or } \alpha_i = \beta_i, \alpha_j = \beta_j, (i \neq j) , \text{ etc.}$$

3 zero eigenvalues $\Rightarrow$

$$\alpha_1 = \alpha_2 = \alpha_3 \text{ and } B = 0 , \text{ etc.}$$

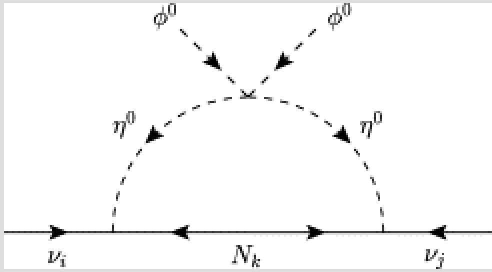
$$B = \sum_{i,j,k} \left[(\beta_i - \beta_j)^2 x_i^2 x_j^2 M_k + 2 x_1 x_2 x_3 (\beta_i - \beta_j)(\beta_j - \beta_k) x_j M_{(7-j)} \right]$$
$$(i, j, k \text{ cyclic})$$

Scotogenic Model

E. Ma, PRD 73, 077301 (2006)

- **Add discrete Z_2 symmetry to the standard model**

  - All SM particles even under Z_2
  - Include RH neutrinos N_k and scalar doublet η, both odd under Z_2
  - Z_2 is unbroken, $< \eta^0 > = 0 \Rightarrow$ Neutrino Dirac mass = 0.
  - N_k Majorana mass is allowed
  - Also allowed is a quartic scalar term: $\lambda_5 (\phi^\dagger \eta)^2$



$$\mathcal{L}_{Yukawa} = f_{ij} \bar{L}_{iL} \phi e_{jR} + h_{ij} \bar{L}_{iL} \tilde{\eta} N_{jR} + h.c.$$

- **One-loop diagram for neutrino mass**

$$(M_\nu)_{ij} = \frac{\lambda_5 v^2}{16\pi^2} \sum_k \frac{h_{ik} hjk}{M_k} \text{ , if } m_\eta \simeq M_k$$

- **Lightest of N_k or *Re* (η^0)/*Im* (η^0) is a dark matter candidate protected by Z_2 symmetry**

Dark Matter

There must be unseen matter in the galaxy extending beyond what is observed.

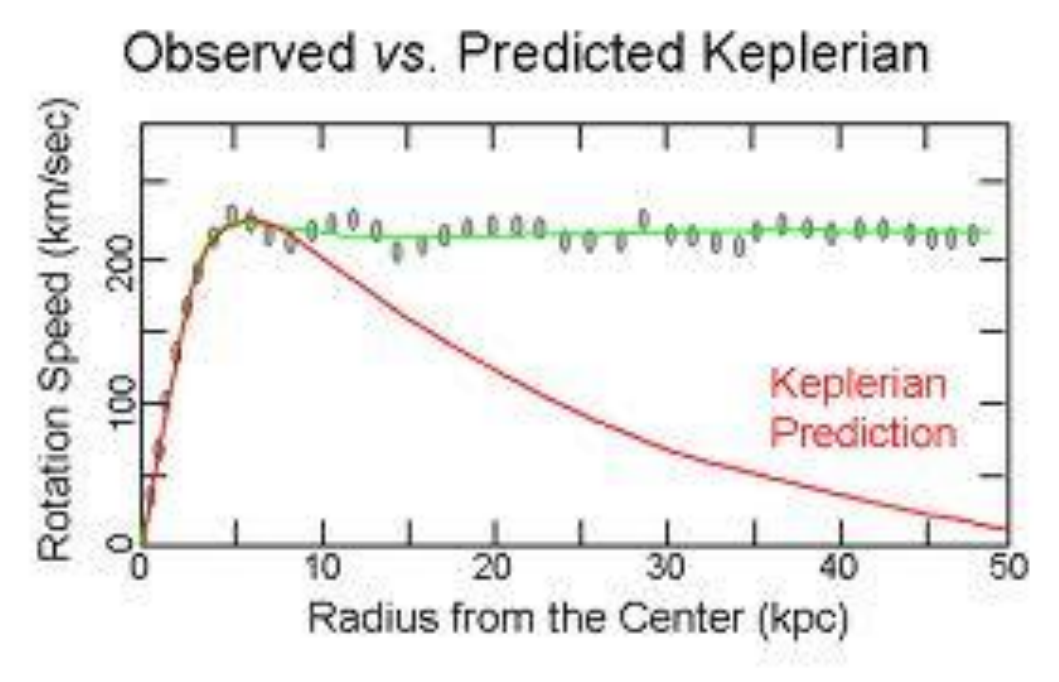
What is this Dark Matter?

Particle physics has several candidates.

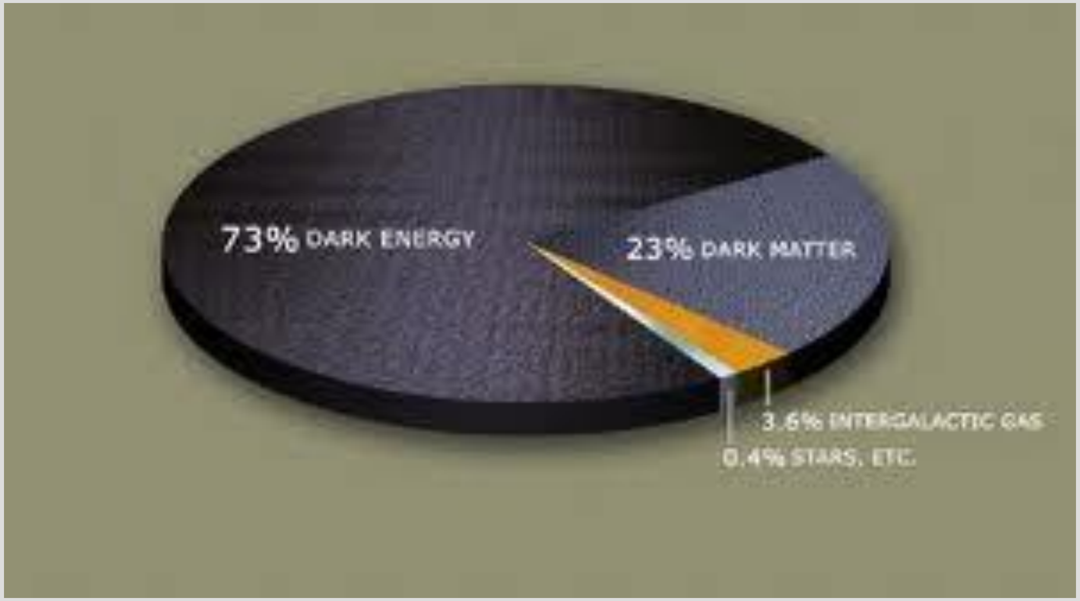
Must be stable and almost Non-interacting

Candidates: lightest SUSY particle
lightest KK-particle, …

Galactic rotation curves



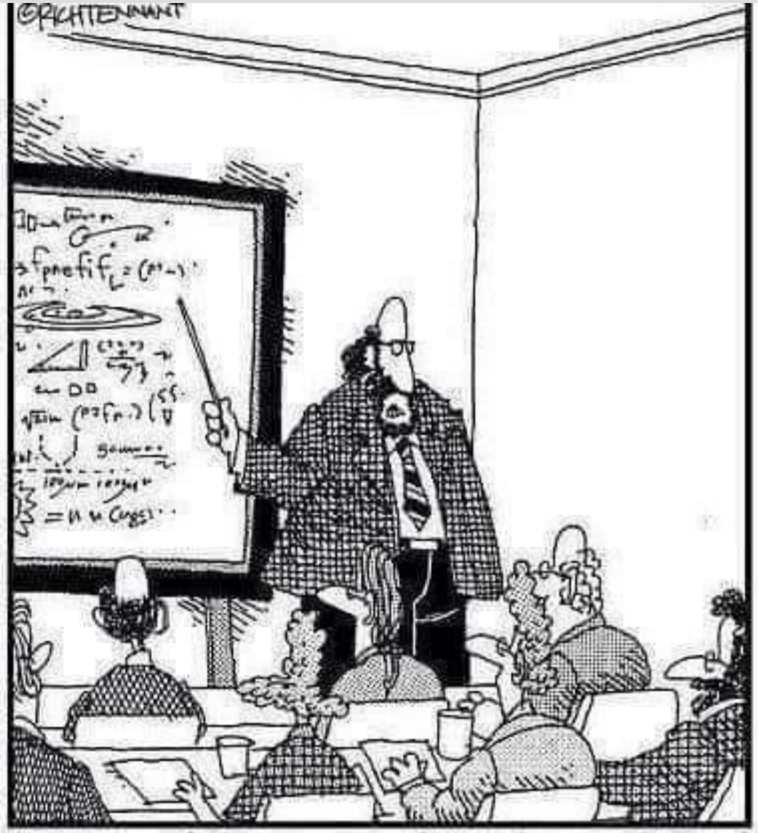
Dark Matter is clumped around galaxies and clusters of galaxies.
Evidence points towards some energy distributed uniformly in all space. Dominant part!
What evidence? The expansion rate of the universe should be slowing down due to gravitational attraction of matter.
Observation: acceleration was less earlier, not more.



Evidence from distant supernovae.
CMBR anisotropy
Structure formation
Particle physics interpretation?

Looking Ahead

- Mass has played diverse roles
- Many issues still open
- Why is the Higgs mass small?
- What is the origin of neutrino mass?
- What is dark matter?
- Many experiments worldwide
- Healthy interplay of astrophysics, cosmology, and particle physics
- New physics: new interactions, symmetries, etc.



"Along with 'Antimatter,' and 'Dark Matter,' we've recently discovered the existence of 'Doesn't Matter,' which appears to have no effect on the universe whatsoever."

Learning from Manoj Banerjee

- Science is continuously evolving

  Keep moving!
- Encourage newly emerging areas
- Theory and Experiment go hand in hand
- Motivate younger people

Keep working hard!

Thank You!