The Interior of Pulsars and issues of Elementarity

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- Let me first thank Professor Bhattacharya and the SINP Alumni Association for inviting me to give this lecture in honour of Professor Manoj Banerjee.
- I have had the pleasure of visiting SINP several times over nearly 5 decades and it is a great pleasure to be back again.
- Throughout those decades I have also been an admirer of Prof Manoj Banerjee and his work.
- I was several years junior to Prof .Banerjee,
- but even as a graduate student in the US, it was a matter of pride for me, as an Indian, to hear my professors talk about the important contributions to nuclear theory that were emanating from India, under the name of one Manoj Banerjee.

- Those early papers of Prof Banerjee , which had already received much international attention, included, among many others,

(i) his classic early papers with Carl Levinson in 1957 on the theory of inelastic p-N scattering using distorted wave Born approximation (DWBA).

(ii) Methods for evaluating shell-model matrix elements in the SU3 classification scheme and their relationship to the rotating nucleus model.

(iii) Shapes of deformed Hartree-Fock intrinsic states such as first excited state of ^{16}O and its symmetries

- But I never had the opportunity of interacting with him at SINP.
- By the time I returned to India after a long stint in the US, Prof Banerjee had done the opposite and moved permanently to the US. !
- However, on the few occasions I had the chance to visit the Univ of Maryland, I would make it point to meet Prof Banerjee to discuss physics and pay my respects.

- Later, Prof. Banerjee, in turn, had moved towards topics overlapping with particle theory, working on a variety of problems including the effect of the the nuclear medium on pion-nucleon scattering, extending the static Chew-Low P-wave description of pi-N scattering, constructing a solitonic solution, which is a linear combination of the nucleon N (I = J = 1/2), and the delta (I = J = 3/2) and so on.
- By that time the center of gravity of my own research had also shifted towards Particle physics . So I was able to keep track of some of Prof Banerjee's later work as well.
- I would, however keep returning from time to time to nuclear systems, not standard areas of nuclear theory, but investigating different conceptual issues.
- The subject of today's talk on Pulsars (Neutron stars) is one such example,
- and it is my honour to dedicate this talk to the memory of Professor Manoj Banerjee

- My choice of this topic and the level of my presentation today has been based on the advice of Prof Sudeb Bhattacharya, that the audience will consist of people from different sub-disciplines and I should not make it too specialised or theory heavy.
- So this talk is not meant for experts in Astrophysics or Nuclear theory.
- I will begin with a general overview of Pulsars , their discovery, and formation . I will also explain why they are examples of Neutron stars.
- Then later I will try to explain some fairly deep conceptual issues that are brought to the fore when thinking about neutron stars.

Neutron stars

- The Neutron star, as the name implies, refers to a giant nucleus of stellar proportions --an entire star of tightly packed neutrons !
- It was hypothesized variously by Lev Landau , Walter Baade and Fritz Zwicky way back in in the 1930's and its theoretical properties studied by Oppenheimer, Tolman, Volkoff ,Salpeter and others.
- It provided a fascinating system for theorists to study nuclear physics in the bulk.
- My own PhD, under Prof Hans Bethe was on such (essentially) infinite nuclear matter.
- But these remained hypothetical objects until the mid 'sixties, when it was realized that the newly discovered stellar bodies called Pulsars were in fact neutron stars

Discovery of pulsars

- The first pulsar was discovered by Jocelyn Bell and Anthony Hewish in 1967. It happened accidentally when they were actually studying distant galaxies. They noticed pulses of radio waves coming from some particular spot in the sky.
- Since then a few thousand have been discovered by the different giant radio telescopes of the world
- Such stars emitting radio pulses were given the name Pulsars (nickname for Pulsating Radio Sources)
- The time interval between consecutive pulses is called the pulsar's *period*.
- Typically pulsars have Periods of the order of a second . For comparison that is the same order as the rate of our heartbeat.
- By now Pulsars have been discovered with periods from a few milliseconds (one millisecond equals 0.001 seconds) up to eight seconds

- The time between pulses is incredibly regular and can be measured very precisely. For example, the pulsar PSR J1603-7202 has been measured to have a period of 0.0148419520154668 seconds.
- In fact in the early days pulses with such amazingly precise period made people wonder if they were being sent by some alien civilisation.
- But soon intense theoretical and experimental analysis gave a comprehensive understanding of pulsars based on purely standard physics principles without invoking alien intelligence or supernatural origins.

Formation of Pulsars in supernova explosions

- In human terms, the creation of pulsars would be a tragic story. They are children born of dying mothers – in this case, stars.
- Stars do die, as will our own sun, someday.
- As you know stars generate their energy through nuclear fusion.
- The fusion reactions give out their energy in the form of neutrons, γ-rays etc. This emitted radiation, generates a huge outward pressure on the star material to counter the inward force of its own gravitational force and keeps the star stable.
- What happens when a star eventually lose all the fuel (Hydrogen , Deuterium..) for fusion?
- Then the star begins to collapse “under its own weight” .

- As the star begins to collapse its atoms get crushed and their electrons are ionised and free to fly around the whole star.
- For simplicity if take the electrons to be free , each will have some momentum p and energy $p^2/(2m)$
- Consider for instance the lowest energy (ground state) of this collection of electrons. (Why only Ground state and not higher energy states? Will be explained later !)
- Because of Pauli Pr, only 2 electrons are allowed for each momentum. As you do for shells of atoms, to get the lowerst energy state, start from p=0 state and keep filling till some p_F.
- In 3 –dimensions, these filled states form a "Fermi sphere" or "Fermi Sea" in p-space of some radius p_F called the Fermi momentum with energy $E_F = (p_F)^2 /2m$

Collapse and stages of stability

Clearly the total number of electrons (= total number of states within that Fermi sphere), will be proportional to the volume of the Fermi sphere. So the electron density will be

$$\rho = N/V \approx p_F^3 \approx E_F^{3/2}$$

- We see that as the star shrinks under gravity, *increasing its density further costs kinetic energy*. This positive energy would offset any gain of potential energy by the gravitational shrinking of the star and further shrinking will stop.
- Depending on the starting mass of the star, this collapse will be halted at different stages.
- Stars like our sun will collapse until they get stabilised at as "white dwarfs", whose size is of the order of that of the Earth.

Incidentally, why did we consider the ground state?

- The filled Fermi sea is only lowest energy (ground) state of that electron gas.
- But isn't the star very hot (White dwarf temp ≈ 100,000 K) ? Should we not consider a thermal mixture of Ground and excited states?
- But as in all physics (as well as in life!), how hot is too hot, is a matter of comparison. For the electron gas in a typical the White dwarf, the Fermi energy comes out to be about E_F ≈0.3 Mev which is equivalent (as kT) to about 3.5 billion degrees.
- (1 MeV = 0.3 x **11,604,525,006.17 kelvin**.)
- So the star's actual temp of 100,000 degrees is really as good as zero at that scale. It will only cause a some excitations near the surface of the Fermi sea and to a good approx, for our talk, you can neglect it.

The Chandrasekhar limit

- What we have described is the fate of the collapse of a star about the mass of our Sun. But, for heavier stars with a stronger gravitational pull, the electronic Fermi pressure may not be enough to prevent further collapse beyond the white dwarf level.
- As was theoretically proved by our own S.Chandrasekhar way back in 1930, if the mass of that resultant white dwarf is $> \approx 1.4$ times the mass of the Sun, its electronic Fermi pressure will not be enough to prevent further collapse due to its own gravity.
- The original Chandra limit was based on just the Fermi gas pressure . A more accurate limit, taking into account e-e repulsion and other effects required more messy complicated calculations.
- Besides, that limit is only for the mass of the White dwarf and not the parent star which collapsed which has to be higher .

- Roughly speaking when stars > about 8 solar masses collapse, they do not stop at the size of a white dwarf (a few thousand km) but continue collapsing .
- Depending on the original mass, when these heavier stars collapse they can they end up as Black holes if they are really heavy (more than, say, 15 solar masses)
- Mentioning black holes, especially in front of general audiences is like dangling a chocolate in front of a child because they dearly love to hear about them. But we will not talk about black holes here
- Instead we will be interested here in stars roughly in the range 8-15 solar masses, whose collapse goes beyond the white dwarf stage, but ends up getting stabilized before it becomes black hole, at a tiny radius of a few kilometres (the size of a small town).
- As the inner part collapses to this tiny size , the energy released makes the outer layer explode outward as reaction to create a spectacular cosmic show.
- That is called a supernova explosion.

The **Crab Nebula** , the remnant of a supernova explosion observed in 1054 by the Chinese, the Arabs…..

As we will show, the small collapsed stars of radius order 10km sitting inside the supernovae are our pulsars.

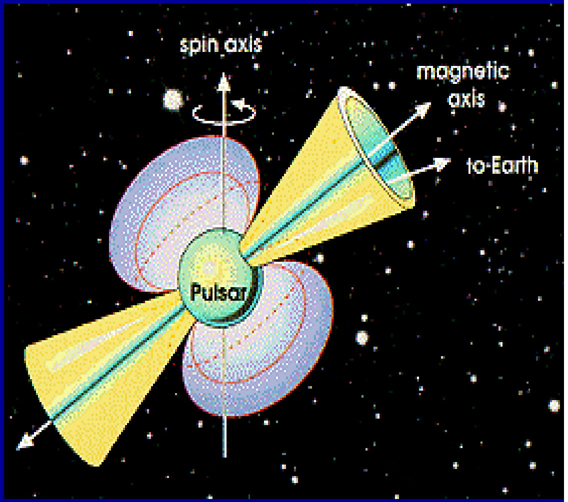
The Crab pulsar is situated near the centre of this nebula and can be “seen” with a radio telescope.

- To understand why that tiny collapsed object is a pulsar, consider what to expect when the original parent star with radius in millions of kilometres collapses to a size of 10 km. This is compressing it by a factor of 10^5 in radius and 10^{15} in volume.
- To start with, atoms are a few angstroms (10^{-10}m) in size, each with a tiny nucleus of radius 10^{-15} m . When this material is crushed by a factor of 10^5 in radius, not only will all the electrons be knocked out, but all nuclei will be squeezed together.
- Then just as valence electrons in metals leave their parent atoms and move all over the metal,
- here the individual protons and neutrons will be liberated from their parent nuclei and start moving to other nuclei, thus forming one giant nucleus!!

- I still have to explain two other things before we move on to deeper conceptual questions:
- 1. How does this neutron Star act as a Pulsar?
- 2. Why call it a “neutron” star when the atoms in the parent star were made of more or less equal proportions of n, p and e ?

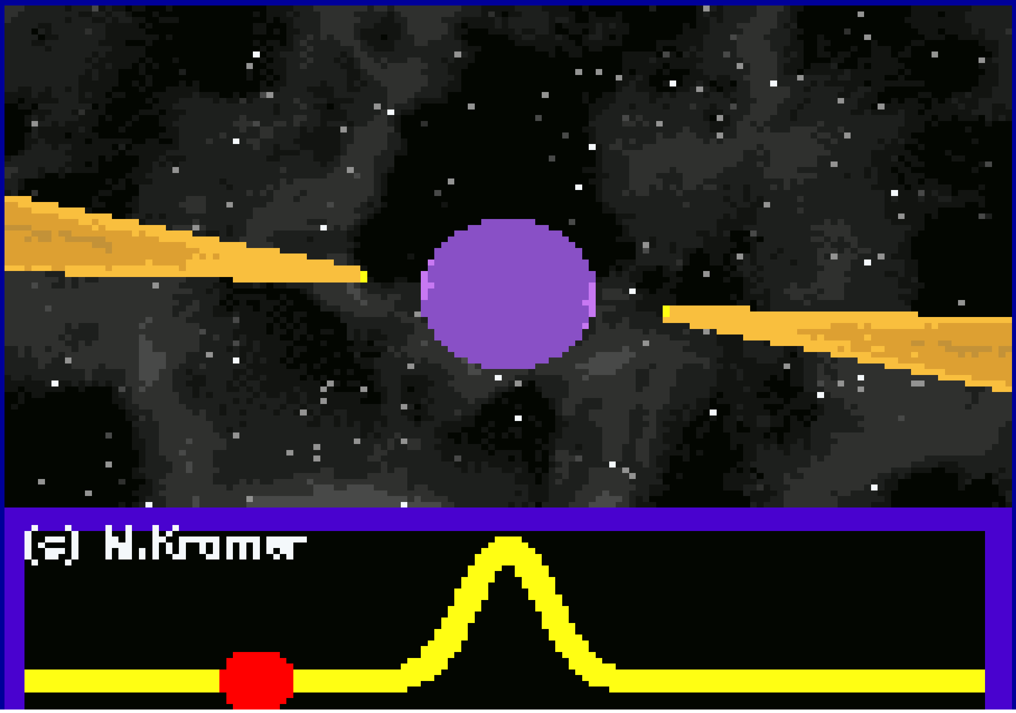
How does it "Pulse" ?

- This collapsed star, now a giant nucleus of size ≈10 km will also inherit some of the angular momentum of the parent star.
- But because the radius has shrunk by 10^5, and moment of inertia by 10^{10}, the angular velocity will be increased by the same factor. It can now rotate at angular velocities of seconds and even milliseconds.
- Finally, like all stars it will have large magnetic fields.
- Recall that our earth also rotates (with a period 24 hours) and has N and S poles of a small magnetic field of about 25 Gauss.
- In the neutron star the B fields are much bigger, ≈10^9 Gauss.
- Once you have such strong B fields, electrons moving in it will radiate e.m. waves as per normal laws of electrodynamics along the magnetic axis



Lighthouse

- As the star is rotating, the emitted beam will also rotate with it . If the earth happens to be along the direction of the emitted beam we will see it as its direction rotates across us.
- The star does not bodily “pulse” like a human heart.
- Rather it is a lighthouse and we are the ships that see it blink as it sweeps across our planet.



Why “neutron star” ? What happened to the protons and the electrons ?

As the star shrinks below the White Dwarf size the relative densities of n, p and e are decided by chemical Equilibrium, As the electron Fermi energy goes shooting up some of them combine with the protons to form more neutrons.

$$n \rightleftharpoons p + e + (\bar{\nu})$$

Fermi Seas of p, e buildup.

Chemical Equilibrium :

$$E_{Fn} = E_{Fp} + E_{Fe}$$

$$M_n c^2 + \frac{P_{Fn}^2}{2M_n} = M_p c^2 + \frac{P_{Fp}^2}{2M_p} + \sqrt{P_{Fe}^2 c^2 + m_e^2 c^4}$$

Recall $\rho = \frac{8\pi}{3h^3} P_F^3$

Charge Neutrality $\Longrightarrow \rho_e = \rho_p \Longrightarrow P_{Fp} = P_{Fe}$

$$\frac{P_{F.n}^2}{2M_n} \approx P_{F.e}\, c \approx 63 \text{ MeV} \Longrightarrow \rho_e = \left(\frac{8\pi}{3h^3}\right)\left(\frac{P_{Fn}^2}{2Mc}\right)^3 = 0.03\,\rho_n^2$$

More precisely $E_{Fe} \approx 61$ MeV

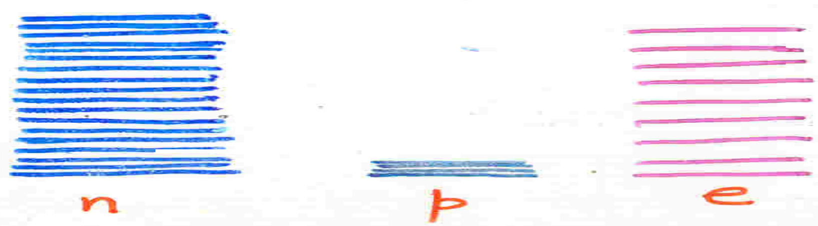
$$E_{F.p} = \frac{P_{Fp}^2}{2M} = \frac{61^2}{2Mc^2} \approx \frac{3700}{1880} \text{ MeV} \approx 2 \text{ MeV}$$

$P_{Fn} \approx 340$ MeV/c

$P_{Fp} = P_{Fe} \approx 61$ MeV/c

$$\rho_p = \rho_e = \left(\frac{61}{340}\right)^3 \rho_n \approx \frac{1}{200}\rho_n$$

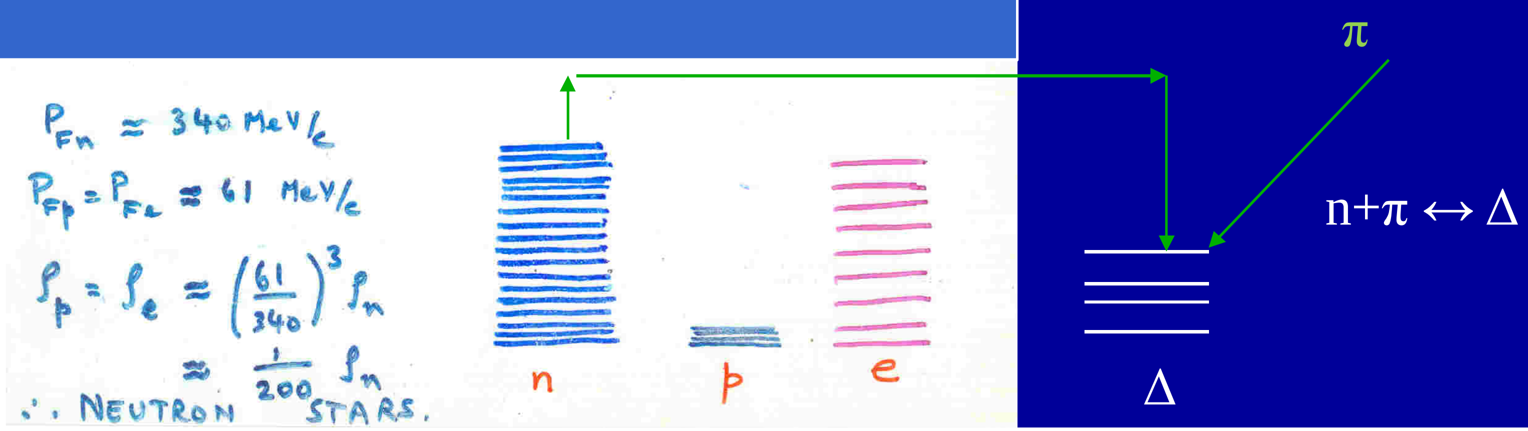
∴ NEUTRON STARS.



- As more detailed calculations of the equation of state of the interior of pulsars were done , it was realized that you could lower the energy further if you could introduce other fermion species with their own Fermi sea.
- So, many people doing neutron star calculations included the Δ resonance as a possible species into their analysis.
- The Δ is created in the scattering of a nucleon with a pion in the J=3/2 , I =3/2 channel as a resonance at a c.m. energy of about 1230 MeV, with a width of 100 MeV
- Like all the ordinary nuclei we are familiar with, the neutron star will also contain virtual pi-mesons, which, along with the nucleons present , can form Δ resonances. So there will be some Δ resonances in the neutron star medium
- If you do treat the Δ as a separate species of particle, then you will certainly lower the energy of the system, as we will see in the next slide.

Thus, including more fermion species lowers the energy cost for further increasing the density. You can “squeeze” the system more easily .The Equation of state is “Softened”

This also started a trend where other particles like the hyperons were also introduced in the mix. .



- This trend of Introducing more and more fermions as elementary species into the system was a welcome development for the nuclear matter community since it generated some more problems for them to do.
- But ,is it correct to treat all these “particles” as separate elementary species in the neutron star system?
- This question belongs to a larger , conceptually more fundamental, and often very confusing subject of what constitutes “elementarity”
- How do we decide which particle is elementary and which is not? What are the independent species of truly elementary particles ?
- Popular writings describe them as the “ultimate constituents of all matter”.
- The goal of finding these ultimate constituents of matter has been a driving force of physics /physical chemistry from the 19th century , eventually culminating in particle physics,
- The idea is based on normal intuition : A city is made of houses, the houses are made of bricks.
- Continuing down, the bricks are made of atoms of Si, Ca, O etc , and finally the atoms were made of electrons and the nucleus and finally the nucleus was made of protons and neutrons.

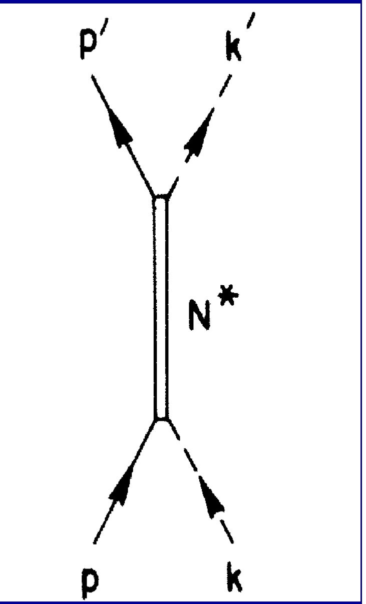
- Up until that stage of nuclear physics, the systems are basically non-relativistic. our classical intuition of constituents and composites continues to hold, appropriately modified by the requirements of quantum theory.
- The constituents are smaller, lighter and found inside the composite
- But at the level of elementary particles, relativity is a crucial ingredient. Combined with strong attractive interactions, it can disrupt our intuitive notions. It cannot be ruled out that composites have the same size and mass as the constituents, (or be even smaller and lighter).
- Remember that as per $E = Mc^2$, a bound system will have less mass than its constituents.
- Also the notion of the “size” of particles, especially resonances , became more hazy, further clouding the distinction between the constituents and composites.
- As a result even though “Elementary Particle Physics” has been pursued for over 70 years there is still no clear definition of an elementary particle !

Returning to the status of Δ in nuclear matter

- On the one hand Gellmann's famous "eightfold way" of classifying Baryons and Mesons, treated the Δ on the same footing as the nucleon and the pion, which were then considered prototype elementary particles
- Yet, it seems absurd to think of a broad resonance like the Δ as a separate elementary particle since it lives for a such a short time (Its resonance width of 100 Mev ≈ 10^{-23} seconds)?
- But we did think of the neutron, the μ and Pi mesons as elementary . They too are not stable . They decay , although after much longer lifetimes than the Δ. But that is only a matter of degree --- not a difference in principle
- Meanwhile, the H atom is very stable but we don't think of it as elementary
- So, clearly there no rigid link between being elementary and being stable, and
- Hence the short lifetime of the Δ cannot be used to rule out its being a distinct

On the other hand, if you think of the Δ as a separate fermion species, then the Δ s will have their own Pauli principle amongst themselves, but no constraint w.r.t. the nucleon, which form a different species.

- That is why we were able to set up a new tower of states for the Δ starting from zero momentum., and that is what led to lowering of energy.
- But that raises other worries
- Isn't the Δ a composite of pi + N since it appears mainly in pi-N scattering as a resonance?
- What about the exclusion principle between the n "inside" the Δ and those "outside" the Δ in the Fermi sea?
- So, a safer way may be to use only pions and nucleons as basic degrees of freedom , and introduce the Δ as a part of the interaction.
- But how do we find an interaction Hamiltonian designed to produce the Delta?
- As you know resonances are generally expressed in terms of scattering amplitudes (S matrices)



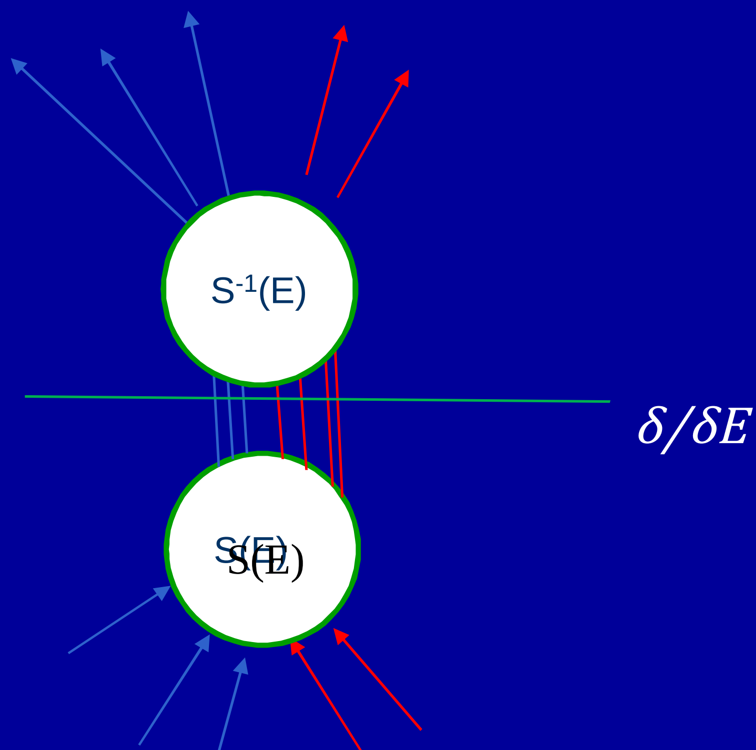
- Clearly the best way to answer such questions for hadron resonances was to work with some S-matrix formulation of thermodynamics, rather than the canonical Hamiltonian formula $Z = Tr [\exp(- H/kT)]$,
- While working on neutron star interiors at Princeton I realized that fortunately such a formulation was available next door!
- An S-Matrix Theory of Stat Mech had recently been constructed by Roger Dashen and Shang-keng Ma whose offices happened to be next door to mine at Princeton
- Both were outstanding theorists who unfortunately died prematurely.
- R. Dashen, S. Ma, and H. J. Bernstein, Phys. Bev. 187, 345 {1969); R. Dashen and S. Ma, J. Math. Phys. 11, 113 {1970);
- R. Dashen and S. Ma, J. Math .Phys. 12, 689 {1971).

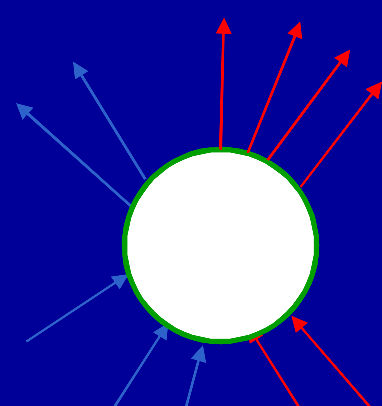
- Starting with the grand canonical formula Z = Tr [exp(- H/kT)] they showed that it can be re-written exactly, to allorders in the density and the interaction, in terms of multi-particle S-matrix elements.

$$\ln Z = \sum_i \ln Z_0^{(i)} + \frac{1}{2\pi i} \sum_{\{n_i\}} \int dE\, e^{-\beta(E-\mu)} \left[\mathrm{Tr}_{\{n_i\}} A S^{-1}(E) \frac{\partial}{\partial E} S(E) \right]_c .$$

- But when expanded it contains a multi-infinite number of terms . After all it contains exactly the full partition function to all orders in the density and interaction.

$$\ln Z = \sum_i \ln Z_0^{(i)} + \frac{1}{2\pi i}\sum_{\{n_i\}}\int dE\, e^{-\beta(E-\mu)}\left[\mathrm{Tr}_{\{n_i\}} A S^{-1}(E)\frac{\partial}{\partial E} S(E)\right]_c .$$



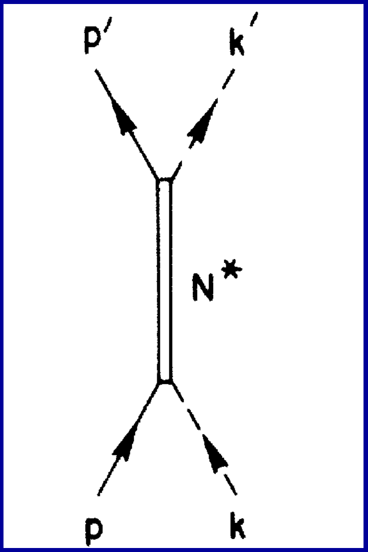


This is a virial expansion, just as in ordinary stat mech formulations. Terms with higher number of legs give the higher virial coefficients. For low densities you can stop with the first few..

- So Dashen and I applied their Master formula to understand the problem of the Δ by modeling it as follows : Consider a boson and a fermion interacting only to the extent of a forming a narrow resonance.
- .R.F.Dashen and R.R., "Narrow resonances in statistical mechanics"Phys. Rev. D 10 , 15 JULY 1974
- The 2-body S matrix will be

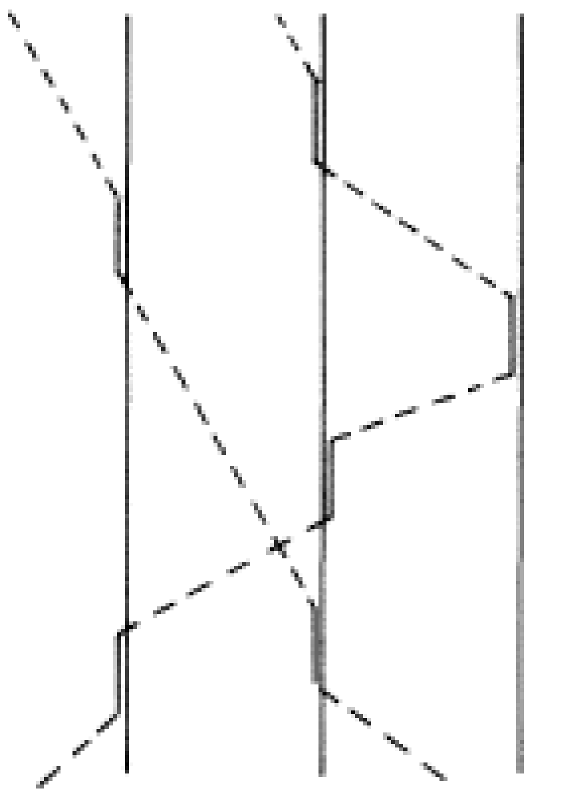
$$\langle p'k'|S|pk\rangle = \langle p'k'|1-2\pi i\delta(e+\omega-e'-\omega')\,T(e+\omega)|pk\rangle$$

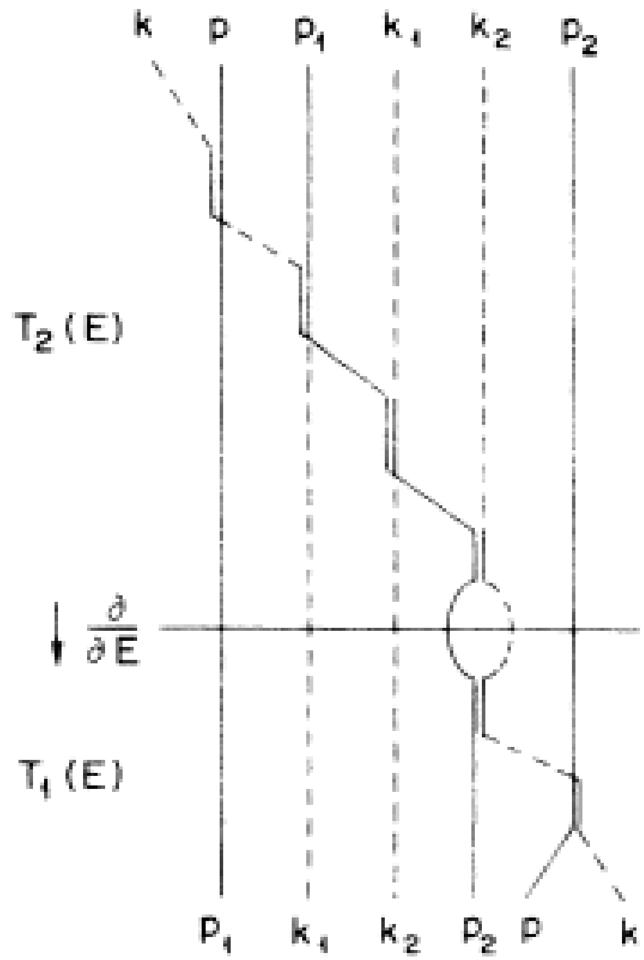
- $$\langle p'k'|T(E)|p,k\rangle = \frac{(2\pi)^3 g^2 \delta^3(\vec{p}+\vec{k}-\vec{p}'-\vec{k}')}{E-[M^2+(\vec{p}+\vec{k})^2]^{1/2}+i\Gamma}$$



This will be the input two body S-Matrix. From this higher body S matrices are constructed, corresponding to such a resonance being created between all pairs of fermions an bosons. and inserted into the Dashen-Ma series So there will be a complicated infinite series of diagrams to sum, which we were able to do exactly.

Examples of diagrams higher order in the interaction and in the density



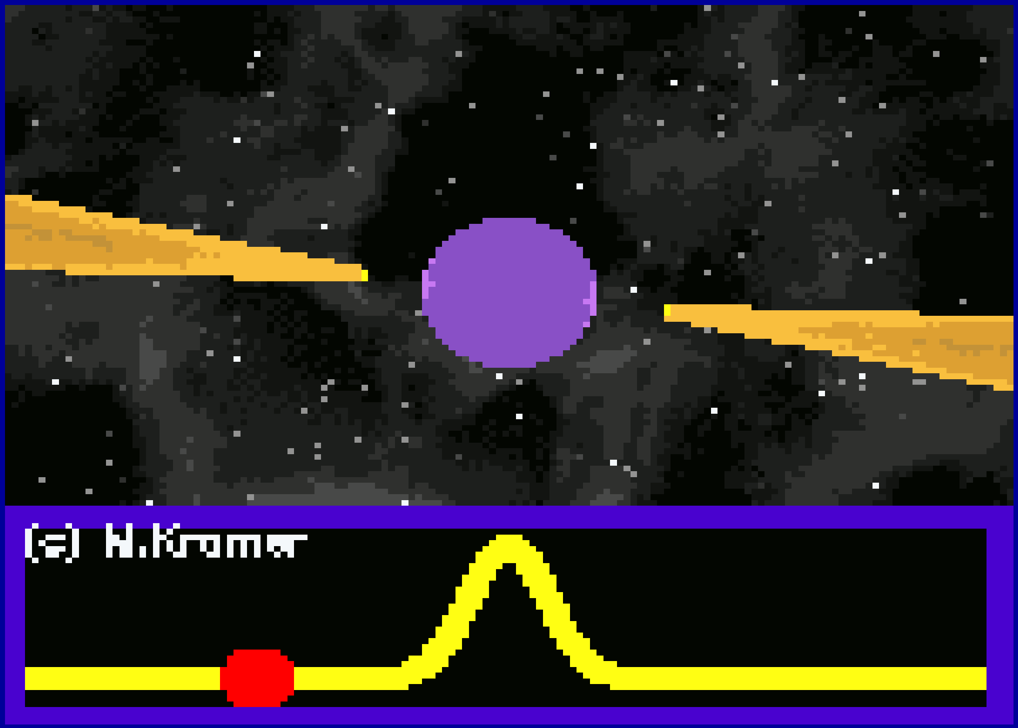


The final result for the exact sum of the entire series becomes

$$\ln Z = \ln Z_0 + V \int \frac{d^3P}{(2\pi)^3} \ln\left(\exp\left\{-\beta\left[(\vec{P}^2 + M^2)^{1/2} - \mu_1 - \mu_2\right]\right\} + 1\right) .$$

- The last term can be recognized as the log-partition function of a another free species of Fermions of mass M ,with chemical potential $\mu_3 = \mu_1 + \mu_2$.
- The 3-3-resonanceΔ , when introduced as part of the S-matrix between the pion and the nucleon effectively acts like a third elementary species under equilibrium conditions n+ $\pi \leftrightarrow \Delta$.
- It doesn't matter whether you introduce it *ab initio* as an elementary species or as a resonance formed during interaction.

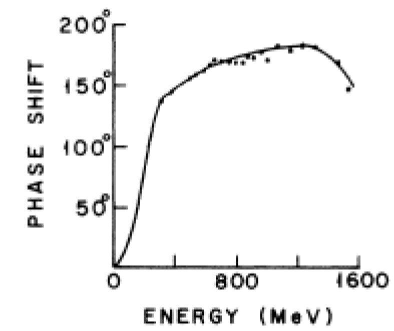
THANK YOU



Consider the lowest virial (2-body) term in the series for Log Z $\approx \frac{1}{2\pi i} \int dE\, S^{-1} \frac{\partial}{\partial E} S$

$S_{\text{two-body}} \approx e^{i\delta(E)}$

$$\frac{1}{2\pi i} S^{-1} \frac{\partial}{\partial E} S = \frac{1}{2\pi} \frac{\partial}{\partial E} \delta(E)$$



$$b_2(P) \simeq \frac{1}{\pi} \int d\epsilon \exp[-\beta(\epsilon^2 + \vec{P}^2)^{1/2}] \frac{\partial}{\partial \epsilon} [\delta(\epsilon)]$$

This is the Beth Uhlenbeck Formula of 1937 for the lowest virial coefficient in terms of scattering data. The Dashen-Ma theory is a generalization of this to all orders in the densities.

A narrow resonance corresponds to $\delta(E) = \pi\ \theta(E-M)$

Then b_2 (P) $= \exp[-\beta(\vec{P}^2 + M^2)^{1/2}]$,

This is just the Boltzmann factor for a relativistic particle of mass M

Why should we care which particles are elementary and which are not??

- When you wish to study the thermodynamics of relativistic strongly interacting systems
- Recall the usual way we do statistical mechanics
- All thermodynamic properties flow from the Grand Partition Function given by the Boltzmann Formula
- $Z = \Sigma[\exp[-(E-\mu)/kT]]$, where E is the energy of a configuration, μ is the chemical potential, and the sum Σ is over all configurations, i.e. all the phase space (or in quantum theory, the complete set of states) of all possible numbers particles of all the species
- In traditional uses of this formula for a mixture of non relativistic species, either classical or quantum mechanical, one knows all the inputs into this formula.
- For , say, a gas mixture of Nitrogen , Hydrogen and Ammonia the sum is over the full phase space (or a complete set of states) of an arbitrary number of N_2, H_2 and NH_3 particles, with a the chemical potential μ reflecting the chemical balance

$N_2 + 3\,H_2 \rightarrow 2\,NH_3$

- Starting with the grand canonical formula $Z = Tr[\exp(-H/kT)]$ they showed that it can be re-written exactly in terms of multi-particle S-matrix elements.
- Most importantly it did not involve knowledge of what the elementary particles are. S matrices of all stable particles were used on the same footing.
- In the short time left for me, I will not attempt to derive their main result. The full proof , with all the corollaries occupies 3 long articles
- I will be content to just the define the the various terms in the formula,
- Then I will quickly discuss how it can be applied in the case of the Δ

- What would the corresponding sum be for an arbitrary dense mixture of sub-atomic particles, especially hadrons, as for instance, study the equation of state of , say, a neutron star ,or the early universe, ?
- The immediate answer would be that you take the sum over all elementary particle species, on the assumption that each species gives rise to independent dynamical degrees of freedom
- That is where we need to know what are the independent species of truly elementary particles
- As I said we don't have a clear general answer to this general question.
- So for specific problems we usually make do with some approximate intuitive choice of what should be considered the independent degrees of freedom
- With this background, let us think about the specific case of the Δ in neutron stars.